

# ERCIM “Alain Bensoussan” Fellowship Scientific Report

**Fellow:** Shalini Gupta

**Visited Location:** FNR Luxembourg

**Duration of Visit:** 21-04-2006 to 31-10-2006

## I - Scientific activity

During my stay at LTI, FNR Luxembourg, I worked with extended finite element method to be used in Fracture mechanics. Initial phase of my stay has been devoted in literature survey and to learn the basic idea of extended finite element method. After a detailed study of the method and the related available research papers, a literature survey report has been prepared. Due to development and propagation of cracks in fracture mechanics, the modeling and computations of evolving discontinuities with the standard finite element method is very difficult and resource consuming. These discontinuities require updating the mesh topology so that it matches the geometry of the discontinuity, which is a cumbersome process. Extended finite element method, which is based on partition of unity method, can be used very efficiently in such problems. This technique allows the entire crack to be represented independently of the mesh and hence re-meshing is not necessary to model crack growth. The essential feature of the method is the incorporation of enrichment functions, which contain a discontinuous field. In the application to fracture mechanics, functions spanning the appropriate near-tip crack field are also included. In X-FEM, the approximation can be written as:

$$\mathbf{u}^h(\mathbf{x}) = \sum_{i \in I} u_i \phi_i + \sum_{j \in J} b_j \phi_j H(\mathbf{x}) + \sum_{k \in K_1} \phi_k \left( \sum_{\ell=1}^4 \mathbf{c}_k^{\ell 1} \gamma_\ell^1(\mathbf{x}) \right) + \sum_{k \in K_2} \phi_k \left( \sum_{\ell=1}^4 \mathbf{c}_k^{\ell 2} \gamma_\ell^2(\mathbf{x}) \right)$$

where  $J$ ,  $K_1$  and  $K_2$  are the nodal sets containing the crack interior and crack tips 1 and 2 respectively.  $\phi_i(\mathbf{x})$  are the standard finite element shape functions for node  $i$  and  $b_j$ ,  $\mathbf{c}_k^{\ell 1}$ ,

$\mathbf{c}_k^{\ell 2}$  ( $\ell = 1, \dots, 4$ ) are nodal degrees of freedom corresponding to the enrichment functions  $H(\mathbf{x})$ ,  $\gamma_\ell^1(\mathbf{x})$  and  $\gamma_\ell^2(\mathbf{x})$  respectively. The Heaviside jump function  $H(\mathbf{x})$  is discontinuous across the crack line and crack-tip functions  $\gamma_\ell^1(\mathbf{x})$  and  $\gamma_\ell^2(\mathbf{x})$  are chosen such that these functions span the near-tip asymptotic fields.

The time was devoted to discuss various technical difficulties which can come while programming the method. After having a clear understanding of the entire method and the necessary subroutines which are required in the method, I have now developed an extended finite element code. The code has initially been tested for a small strain linear elasticity problem to model the crack growth. In future collaboration the work will be extended for hyper-elastic materials and other complex conditions.

## **II- Publication(s) during your fellowship**

In preparation

## **III -Attended Seminars, Workshops, and Conferences**

Attended the Workshop on High Order Methods for Large Scale Industrial Applications, 6th-9th June, 2006 at CRS4, Pula(CA), Italy.