ERCIM "Alain Bensoussan"
Fellowship Scientific Report

Fellow: Stelios Charalambides
Visited Location: NTNU (Norwegian University of Science and Technology)
Duration of Visit: 12 months

I - Scientific activity
(1 page at maximum)

During my time at NTNU I worked on two areas, torsion theory and representation theory.

In the torsion theory area I worked on the article "τ-Uniform modules" which was based on previous work. It was completed during the fellowship and presented at a conference. I also did some minor revisions on two articles which I had submitted before the start of my fellowship. They had been co-authored with John Clark during my PhD studies. One of them was published in the Journal of Algebra and its Applications in 2008 under the title of "Max modules relative to a torsion theory". The other, entitled "τ-Injective modules", appeared in Modules and Comodules, Trends in Mathematics, again in 2008.

In Representation Theory, which is the area of main interest at NTNU, my objective was to get acquainted with the main concepts and techniques and find links to Torsion Theory. Towards this goal I worked as follows.

First I had to familiarize myself with the background material from Category Theory and Homological Algebra. I looked at several sources mainly [W], [ASS], [RS] and [AF] in consultation with my scientific co-ordinator. In parallel I worked on the concepts of J-modules, quivers and algebras from [RS] and[ASS]. Furthermore I audited a course focusing on almost-split sequences and the Auslander-Reiten quiver based on [ARS].

I also joined two working seminar groups on "Coverings in Representation Theory" and "Representation Dimension". The first was already going when I joined NTNU in November 2007 and kept on until December 2007. The second was running from February to April 2008. Moreover, I attended the weekly algebra seminars at NTNU. There I had the opportunity to listen to many interesting talks from local people and international visitors on various branches of Representation Theory. Finally I participated in a Summer school on "Group actions in geometry and representation theory".

During most of my stay I had weekly meetings with my scientific co-ordinator. We discussed the various questions that arose from my study, the talks and the seminars that I had attended.

[AF] Rings and Categories of Modules (Anderson, Fuller)
[ARS] Representation Theory of Artin Algebras (Auslander, Reiten, Smalø)
[ASS] Techniques in Representation Theory vol. 1 of Elements of the Representation Theory of Associative Algebras (Assem, Simson, Skowroński)
[RS] Introduction to Representation Theory of Algebras (Ringel, Schröer)
[W] An Introduction to Homological Algebra (Weibel)

II- Publication(s) during your fellowship

Please insert the title(s), author(s) and abstract(s) of the published paper(s). You may also mention the paper(s) which were prepared during your fellowship period and are under reviewing.

Title: Max modules relative to a torsion theory.
Authors: Stelios Charalambides and John Clark.
Publication: J. Algebra Appl. 7 (2008), no. 1, 21--45.
Abstract: We introduce the concepts of τ-Max modules and left τ-Max rings which are torsion-theoretic analogues of Max modules and left Max rings. A generalization is obtained of an important theorem by Shock and used to characterize τ-Noetherian rings using τ-Max modules. We then characterize left τ-Max rings and obtain a torsion-theoretic version of a result by Hirano. We conclude with results on τ-short modules, introduced as a torsion analogue of a concept recently defined by Bilhan and P.F. Smith.

Title: τ-Injective modules.
Authors: Stelios Charalambides and John Clark.
Publication: Modules and Comodules, Trends in Mathematics, Birkhauser (2008), 143-168.
Abstract: In this article we consider injective modules relative to a torsion theory τ. We introduce τ-M-injective and s-τ-M-injective modules, relatively τ-injective modules, the τ-M-injective hull and Σ-τ-M-injective and Σ-s-τ-M-injective modules. We then examine the relationship between these new concepts and the ones in the literature.
Some of the new results obtained include a Generalized Fuchs' Criterion characterizing s-τ-M-injective modules and a Generalized Azumaya's Lemma characterizing τ-$\oplus_I M_i$-injective modules. We also give a proof of the existence and uniqueness up to isomorphism of the τ-M-injective hull. Furthermore we provide generalizations of results by Faith, Albu and Năstăsescu which characterize Σ-τ-injective and Σ-s-τ-M-injective modules. We conclude with a torsion theoretic version of a result of Cailleau which gives a condition for a direct sum of Σ-s-τ-M-injective modules to be Σ-s-τ-M-injective.

Title: τ-Uniform modules.
Authors: Stelios Charalambides.
Publication: (to be submitted to the "Rings and Modules" conference proceedings in Lisbon).
Abstract: In this article we introduce a new concept, that of a w-τ-uniform module. It generalizes the known concept of a τ-uniform module which is a special case of a uniform module. In the outset we elaborate on the properties and relationship between w-τ-essential submodules and w-τ-complements (first introduced by Pardo in [GP]). We then use w-τ-essential submodules to define w-τ-uniform modules and study their properties. Using the concept of a w-τ-uniform module we obtain an alternative characterization of the τ-Goldie (τ-uniform) dimension. We then try to determine which of its main features carry over to the torsion theoretic setting. We conclude with relative versions of a result by Camillo that characterizes quotient finite dimensional modules, and a result by Shock which characterizes noetherian modules using the concepts of Max modules and the uniform dimension.

[GP] J. L. Gomez-Pardo, Spectral Gabriel Topologies And Relative Singular Functors, Comm. Algebra 13 (1985), no. 1, 21–57.

III -Attended Seminars, Workshops, and Conferences

Please identify the name(s), date(s) and place(s) of the events in which you participated during your fellowship period.

Name: Algebra Seminar.
Dates: 29th October 2007.
Place: Charles University, Prague, Czech Republic.

Name: Algebra Seminar.
Dates: Weekly: 5th November 2007 – 15th May 2008 and 28th August – 2nd October 2008.
Place: NTNU, Trondheim, Norway.

Name: Coverings in Representation Theory Seminar.
Dates: Weekly 2nd November – 6th Decemer 2007.
Place: NTNU, Trondheim, Norway.

Name: Representation Dimension Seminar.
Dates: Weekly 19th February – 14th April 2008.
Place: NTNU, Trondheim, Norway.

Name: SUMMER SCHOOL: Group actions in geometry and representation theory.
Dates: 16th – 20th June 2008.
Place: Sophus Lie Conference Center, Nordfjordeid, Norway.

Name: Rings and Modules conference in honour of Patrick F. Smith's 65th birthday.
Dates: 16th – 19th September 2008.
Place: University of Lisboa, Lisbon, Portugal.

Name: Algebra Seminar.
Dates: 9th October 2008.
Place: Warsaw University, Warsaw, Poland.

IV – Research Exchange Programme (12 month scheme)

Please identify the name(s), date(s) and place(s) of your Research Exchanges during your fellowship period and detail them.

Name: CRCIM, Charles University.
Dates: 17th - 31st October 2007.
Place: Prague, Czech Republic.
Details: I gave a talk at the Algebra Seminar on the 29-10-2007 on "Uniform Modules Relative to a Torsion Theory". I also had a series of meetings and discussions with Jan Trlifaj about getting a torsion theoretic version of Goldie's Theorem.

Name: PLERCIM, Warsaw University.
Dates: 5th-17th October 2008.
Place: Warsaw, Poland.
Details: I will be giving a talk on "Injective modules relative to a torsion theory" on the 9-10-2008. I will also have meetings with Professors Jan Krempa, Edmund Puczyłowski and Jerzy Matczuk to discuss various aspects of the uniform dimension in the lattice and torsion theoretic settings.