ERCIM "Alain Bensoussan" Fellowship Scientific Report

Fellow: Agisilaos Athanasoulis
Visited Location : INRIA
Duration of Visit: November 2007 – October 2008

I - Scientific activity

The topic of this project has been the further development of the smoothed Wigner transform and its application, mainly in semiclassical problems. The outlook and the work undertaken have both theoretical and numerical components. Here we will focus mostly in the work *completed* so far, which is more of theoretical nature.

The body of the completed work is contained in "Coarse-scale representations and smoothed Wigner transforms", a paper submitted for publication at the *Journal de Mathématiques Pures et Apliquées* which was recently accepted – technically "to accepted be upon the implementation of the reviewer's suggestions"', as is customary. A preprint can be found online at http://hal.inria.fr/inria-00320985/en/.

The first thing that is addressed is the formulation of an appropriate functional analytic framework. Indeed, smoothed Wigner equations (and more generally "smoothed equations», defined precisely in the paper) involve operators of infinite order, and therefore the usual machinery of Schwarz distributions, pseudodifferential operators of finite order etc. is not sufficient even for their formulation. The key observation is that smoothed functions belong in certain entire-analytic classes, related to Gelfand-Shilov spaces of type $\mathcal{S}^{\beta}$. In this framework, the "smoothed calculus" is formulated: for any system of linear equations, the exact equations governing a smoothed version of the solution of the original system are derived. The exact equations for smoothed Wigner transforms follow as an application. It must also be noted that for certain nonlinear Schrödinger equations, the smoothed Wigner equations can still be formulated (without any deconvolutions). (Previously the smoothed Wigner equations had only been formulated explicitly for problems with polynomial Weyl symbols, in earlier work by the fellow).

Once the equations are formulated, we proceed to show a first application in the study of semiclassical limits. Indeed, semiclassical limits have been the main motivation for this work, and it is where the concept of the smoothed Wigner transform (sometimes also called Husimi transform) originates. We show that for a natural scaling of the smoothing in the semiclassical parameter e, we can have stronger error estimates in the regime $\varepsilon \to 0$. In particular we can have pointwise convergence, while with the Wigner-transform-based state of the art the convergence is in a much weaker sense. This is related to one of the main objectives of this research program, i.e. showing that semiclassical convergence results can be improved / strengthened by the use of the smoothed Wigner transform.

Other activities include:

- In relation to this project, a thematic program has been co-organized in the Wolfgang Pauli Institut (WPI). In particular numerical work in collaboration with colleagues from

Vienna is in progress. Two workshops have also been organized (one held, one to be help); for more details see section IV.

- The investigation of semiclassical limits in Schrödinger equations with "singular potentials". The state of the art is that semiclassical limits exist for C^1 potentials, and in general are uniquely defined for $C^{1,1}$ potentials. There is some good evidence that using the SWT both these thresholds can be lowered. This is still in progress, in collaboration with Prof. T. Paul of the DMA, ENS.

II- Publication(s) during your fellowship

"Coarse-scale representations and smoothed Wigner transforms", A.G.A., N. Mauser and T. Paul, 58 pages, accepted for publication in the *Journal de Mathématiques Pures et Apliquées*

III -Attended Seminars, Workshops, and Conferences

The work carried out during the ERCIM fellowship has been presented in several seminars, workshops and conferences, including:

- ***Groupe de Travail ANALYSE NON LINEAIRE***, Université Pierre et Marie Curie – ENS, April 9 2008
- ***"Classical and Modern Harmonic Analysis: from Theory to Numerical Computation"***, April 30 - May 2 2008, Edinburgh, UK, website: http://www.univie.ac.at/nuhag-php/icms08/
- ***"12th Panhellenic Analysis Conference"***, May 15 – 17 2008, Athens, Greece, website: http://users.uoa.gr/~apgiannop/pcma08/index.html
- ***"12th International Conference on Hyperbolic Problems: Theory, Numerics, Applications"***, June 9 – 13 2008, Maryland USA, website: http://hyp2008.umd.edu/
- ***Seminaire Laboratoire Jacques-Louis Lions***, Université Pierre et Marie Curie, June 27 2008

Moreover, during this time, and in relation to the work supported by this fellowship, two workshops have been co-organized by the fellow at the Wolfgang Pauli Institute; one which took place in September, http://www.wpi.ac.at/event_view.php?id_activity=108, and one which is to take place in November. See also next section for more details.

IV – Research Exchange Programme (12 month scheme)

The nodes for the research exchange program were the WPI in Vienna; scientific contact N.J. Mauser <norbert.mauser@univie.ac.at>; and FORTH in Heraklion; scientific contact George Makrakis <makrakg@iacm.forth.gr>.

The fellow visited Heraklion in September 2007, and also had a chance to meet with Prof. Makrakis in Vienna in the "Applied Analysis and Fast Computation in Phase Space" workshop, in September 2008. Several weeks were spent in Vienna, including in late January – early February 2008, in July 2008, and of course in September 2008 for the aforementioned workshop.

The workshop itself is part of a larger thematic program, http://www.wpi.ac.at/theme_view.php?id_theme=33, co-organized by the fellow, the main objectives of which are the communication with colleagues active in related fields (through the workshops), and the numerical application of these ideas. In this context, collaboration with several colleagues in Vienna is in progress on the development of a parallel code for the implementation and evolution of the smoothed Wigner transform.