# ERCIM "Alain Bensoussan"
# Fellowship Scientific Report


Fellow: Dr. Christian Schellewald
Visited Location : Department of Electronics and Telecommunications (IET)
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Fellowship Duration: 1st March 2009 to 28th February 2010

## I - Scientific activity

Within my ERCIM-Fellowship I was associated to the research team of Prof. Ralf Müller in the Signal Processing Group at the Norwegian University of Science and Technology (NTNU). The research stay allowed me to familiarise with state-of-the-art solution methods for problems that arise in signal processing and transfer their mathematical model formulations and solution algorithms to problems in the field of computer vision that arise for example in object recognition tasks.

In order to explore the applicability/usability of belief propagation algorithms to a combinatorial optimisation formulation of graph matching problems I introduced a probability measure that incorporates a combinatorial energy-functional and allows the decomposition of this measure into appropriate factors. Note that maximising the probability measure is equivalent to minimising the energy-functional. Exploiting this reformulation the structure of the associated factor-graph can be extracted and results -- apart from the variable nodes -- in four different types of factor-nodes in the factor-graph which require different computations to obtain the appropriate outgoing messages. However, the size of the factor-graph itself grows quickly with the size and number of edges in the graphs of the original matching problem. A straight forward implementation lead, even for small problem sizes, to quite high computation times. Exploiting a Log-Likelihood-Ratio formulation within the factor-graph algorithms and exploring different computation schedules enabled us to cope with larger problem sizes. Our preliminary results were promising and in order to obtain a better understanding and insight to the factor-graph approach I reduced the full problem to a problem that is known to be efficiently solved to optimality by linear programming. Experiments showed that the max-sum algorithm for the reduced problem lead to the optimal solutions as well. This has to be investigated theoretically and in more detail in the future.

In connection with a convex relaxation bound for a subgraph isomorphism problem I investigated the relation of a subgraph matching formulation with the maximum clique approach for finding a maximum common subgraph. I found that the additional knowledge (in particular the bipartite matching constraints) that is present in my problem formulation can be exploited to obtain an improved upper bound (for the clique size) for the equivalent max-clique formulation of the problem.

In addition to the topics above I submitted a funding application for in the field of computer vision to the Research Council of Norway. (The results are expected to be published in April 2010.)

## II- Publication(s) during your fellowship

*Title: Convex Relaxation Bound for Non-Subgraph Isomorphism*
*Author: Christian Schellewald*
*Abstract:*
*In this paper a convex relaxation of a subgraph isomorphism problem that leads to a new lower bound is proposed. This bound can provide a proof that no subgraph isomorphism between two graphs can be found. The computation is based on the SDP relaxation of an elegant combinatorial optimisation formulation for subgraph isomorphism. The combinatorial optimisation formulation and its convex relaxation is explained in detail. We consider subgraph isomorphism problem instances where only the structures of the two graph instances are given which means that we deal with simple graphs in the first place. The idea for the bound is based on the fact that a subgraph isomorphism for such problem instances always leads to zero as lowest possible optimal objective value for our combinatorial optimisation problem formulation. Therefore a lower bound that is larger than zero represents a proof that a subgraph isomorphism can not exist in the problem instance. But note that conversely, a negative lower bound does not imply that a subgraph isomorphism must be present and only indicates that a subgraph isomorphism is still possible. Furthermore an interesting relation of our approach to the maximum clique problem is presented.*
*Status: Under Review.*
*S*ubmitted to the *Computer Vision and Image Understanding Journal (19 pages).*

## III -Attended Seminars, Workshops, and Conferences

*IEEE Information Theory Winter School. March 29 to April 3, 2009 in Loen, Norway*

*I attended and presented my research work at the irregular scheduled seminars of the Department of Electronics and Telecommunications at the NTNU.*

## IV – Research Exchange Programme (12 month scheme)

***First Research Exchange Institute:*** *Institute for Computer Graphics and Vision (ICG) at the* Graz University of Technology, Austria. ($11^{th}$ to $20^{th}$ January 2010)

*The ICG research group has about 80 members and is focused on Computer Graphics, Visualization, Medical Computer Vision, Object Recognition, Object Reconstruction, Robotics, Virtual Reality and Augmented Reality. During the visit I had the opportunity to present my research work to the ICG group. The discussions with several group-members brought interesting algorithms for solving inverse problems in signal/image processing to my attention. Furthermore I had the opportunity at the Institute to get insights and to discuss state-of-the-art 3D Reconstruction methods and optical flow computer vision algorithms that were implemented on the GPU (the floating point processing unit attached nowadays to some graphics cards allowing a high parallelisation of time consuming computations).*

***Second Research Exchange Institute:*** *The Image group at DIKU (the department of Computer Science at the University of Copenhagen) in Denmark, ($8^{th}$ to $12^{th}$ February 2010)*

*The Image group consists of about 40 researchers doing research in the fields of Image Processing, Computer Vision, Robotics, Statistics, Computer Graphics, High Performance Computing, and Data Collection.*
*The exchange provided me with the opportunity to present my work and to have distinct meetings and discussions with many of the researchers employed in the Image group. This provided me with insights about new photorealistic rendering techniques, approaches in tree-matching, the extraction of non-rigid structures from image sequences using prior knowledge along with practical optimisation methods that are used within the group.*