

ERCIM “Alain Bensoussan” Fellowship Scientific Report

Fellow: SHAHAF (HAHAMOV) NITZAN
Visited Location : NTNU, Trondheim, Norway
Duration of Visit: 1/3/2009 – 30/6/2010 (until 28/2/2010 under ERCIM Fellowship)

I - Scientific activity

My work at NTNU has focused on the general area of systems of vectors in Banach function spaces: the basic building tools for expansions/approximations of functions. This subject lies at the intersection of several fields in analysis, including approximation theory, time-frequency analysis, Fourier analysis, spaces of holomorphic functions, and the geometry of Banach spaces.

One application of this subject is the modern area of time-frequency and Gabor Analysis, whose implications range from the study of pseudo-differential operators to the analysis of music signals. Questions in this area regard the optimal time-frequency localization possible for a generator of a "good" system, and the density of the corresponding time-frequency translations of this generator. Together with J.-F.Olsen, a doctoral student with Professor Seip at NTNU we studied Balian-Low type theorems for exact systems, with a frame type approximation property. This allowed us to continuously interpolate the Balian-Low theorems for Riesz bases and the corresponding theorems for systems which are merely exact. To the best of our knowledge this is the first result which describes a continuous transition from exact systems to Riesz bases. A paper based on this work, coauthored by J.-F.Olsen and myself was submitted to JFAA (Journal of Fourier Analysis and Applications). It is important to mention that the subject of our paper and of the literature that precedes it is relevant to other areas of interest in applied mathematics include compressed sensing, a subject connected to the question of universal sampling discussed above: One wants to give a "very good" reconstruction using "very sparse" samples, at least with high probability.

A second project, about to be concluded, was conducted in collaboration with J. Marzo, a postdoc in Seip's group and J.-F. Olsen. In this collaboration we studied systems of reproducing kernels in De Branges spaces with a phase function that gives a doubling measure. The study of systems of vectors is realized within the research on spaces of holomorphic functions. In such spaces, one considers systems of reproducing kernels whose properties correspond to the sampling and interpolation possibilities of certain sequences in the complex plane. Here, there is a vast amount of different settings to consider, each one presents a different type of problems. Of particular interest to me are the de Branges spaces (connected to the so called model spaces), which have recently become a focus of attraction. Their study is connected to certain questions in mathematical physics which make it reasonable to assume that they will become a subject of study also among applied mathematicians. In our work we have studied quite a large family of such spaces, and described how most of the classical results about exponential systems manifest in this setting.

My future plans, as I see them now, is to expand the different settings I consider in my work to wavelet theory and compressed sensing on the one side, and to spaces of holomorphic functions, polynomial spaces, and higher dimensional spaces on the other. I plan to engage in this activity during in my next postdoctoral position, at the University of Missouri -Columbia,

II- Publication(s) during your fellowship

(1) S. Nitzan and J.-F. Olsen,
From exact systems to Riesz bases in the Balian-Low theorem,
Submitted for publication to JFAA (Journal of Fourier Analysis and Applications),
arXiv:0906.2302, (2009).

(2) J. Marzo, S. Nitzan and J.-F. Olsen,
Sampling and interpolation in certain de Branges spaces,
In Preparation.

III -Attended Seminars, Workshops, and Conferences

Active participation in meetings – gave talks in the following meetings:

Conference on Time-Frequency Analysis, 2009, Strobl, Austria
Norwegian Fall School on Analysis, 2009, Steinkjer, Norway

IV – Research Exchange Programme (12 month scheme)

21.6-27.6: A visit to the WPI (Wolfgang Pauli Institute) in Vienna

During this visit, I had the chance to visit also the Nuhag group at the University of Vienna, and was fortunate to start several collaborations which I am still perusing today. During this visit I also participated and gave a talk in a the Strobl meeting described above under III.

27.6-10.7: A visit to the UPC (Universitat Politècnica de Catalunya) in Barcelona

This visit was very productive for me. Indeed, problems that I became familiar with during this visit are currently my main research activity.