ERCIM "Alain Bensoussan" Fellowship Scientific Report

Fellow: Sergiy Zhuk
Visited Location : INRIA Paris-Rocquencourt research center
Duration of Visit: 12 month

I - Scientific activity

The subject of the scientific activities was to develop the minimax a posteriori motion estimation algorithm for the 2D incompressible Navier-Stokes and Euler equations with uncertain parameters given a sequence of images displaying the motion of an incompressible fluid flow.

The scientific activities consisted of the following three points:

1) theoretical development of the a posteriori minimax state estimation approach for 2D incompressible Navier-Stokes and Euler equations;
2) development of the reduction procedure;
3) implementation of the resulting algorithm in the generic data assimilation library Verdandi

In the point 1), I worked on the existence and uniqueness of the viscosity solution of the Hamilton-Jacobi-Belman (HJB) equation in Hilbert space. This solution is a value function of the optimal control problem with quadratic cost and Navier-Stokes Equation (NSE) as a constraint.
The result in this direction is a proof of the existence and uniqueness of the viscosity solution of the HJB in finite dimensional space, which is the value function of the finite dimensional optimal control problem with quadratic cost and finite dimensional Galerkin approximation of the NSE as a constraint. This value function, in turn, is shown to approximate (in the point-wise sense) the value function of the original infinite-dimensional optimal control problem.
The sub-level set of the value-function describes the reachability set of NSE. The reachability set is compatible with observations and uncertainty description. It represents a set-valued estimation of the velocity field associated with an observed motion field. The minimax estimate of the velocity field can be set, in particular, to the Tchebysheff center of this set-valued estimation.

In the point 2), the result is a robust reduction procedure for the NSE based on the Principal Component Analysis (PCA). The procedure consists of the generation of the extended set of snapshots, which represents different possible configurations in the state space of the 2D NSE with uncertain but bounded initial velocity field. The robust reduced basis is then generated by means of the PCA applied to the extended set of snapshots. The robust reduced model is obtained by projecting the NSE onto the reduced basis. The main advantage here is a low dimension of the state space of the reduced model.

In the point 3), the result is an implementation of the minimax state estimation algorithm in the generic data assimilation library Verdandi.

II- Publication(s) during your fellowship

1. S. Zhuk, V. Mallet, Data assimilation with a reduced minimax approach in a chemistry-transport model, INRIA research report, 2010
2. S. Zhuk, I. Herlin, A posteriori minimax state estimations for 2D Navier-Stokes equations, INRIA research report, 2010
3. S. Zhuk, Inverse problems for linear ill-posed differential-algebraic equations with uncertain parameters, submitted to Proceedings of the 8th AIMS Conference on Dynamical Systems, Differential Equations and Applications, 2010

III -Attended Seminars, Workshops, and Conferences

1. The ADAMS Project Meeting, INRIA, Paris, 2009 . ADAMS is an associate team of INRIA with partners in Georgia, Ukraine and Russia.
2. The Geo-FLUIDS project meeting, INRIA, Rennes 2010. Geo-FLUIDS is an ANR project, led by INRIA project-teams FLUMINANCE, CLIME and MOISE.
3. The 21st International Workshop on Operator Theory and its Applications, Berlin, Germany, July 2010
4. The 8th AIMS Conference on Dynamical Systems, Differential Equations and Applications, Dresden, Germany, May 2010

IV – Research Exchange Programme (12 month scheme)

1. FORTH IACM, May 3-7, 2010

Name of the scientific contact: Dr. Nikos Kampanis, Principal Researcher at the Numerical Analysis, Computational Fluid Dynamics and Scientific computing Group.
Description: I gave a talk at the scientific seminar of the group. The main topic was a computational aspects of generation of the robust proper orthogonal decomposition for 2D Navier-Stokes equation.

2. CWI, August 23-28, 2010

Name of the scientific contact:Jason Frank, Leader of the Dynamical Systems and Numerical Analysis Group. Description: I gave a talk at the scientific seminar of the group. The main topic was data assimilation with minimax state estimation approach for 2D Euler equation.

Moreover, discussions with specialists in numerical analysis and computational fluid dynamics allowed me to better understand the algorithmic side of the data assimilation problem for the 2D incompressible fluids. In addition, I found several interesting applications for the minimax state estimation framework, namely improving of the quality of the robust PCA with help of observed sequence of images.