ERCIM "ALAIN BENSOUSSAN"
FELLOWSHIP PROGRAMME

ERCIM
European Research Consortium
for Informatics and Mathematics

Scientific Report

First name / Family name	Michal Garlik
Nationality	Czech Republic
Name of the *Host Organisation*	University of Warsaw - Faculty of Mathematics, Informatics, and Mechanics
First Name / family name of the *Scientific Coordinator*	Leszek Kolodziejczyk
Period of the fellowship	01/10/2015 to 30/09/2016

I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

The most fruitful area that I worked on with my scientific coordinator during my one year fellowship period turned out to be the proof complexity of bounded depth Frege proof systems with the parity connective. We considered three relatively strong families of subsystems of the above systems and we compared these systems in terms of proof sizes of formulas in De Morgan language as well as formulas in De Morgan language together with the parity connective. We obtained both simulation and separation results for our families of systems. The results of this work were presented by L. Kolodziejczyk in an invited talk at the Proof Complexity Workshop in St. Petersburg (organized as part of the Special Semester Program on Complexity Theory). Our work has lead to a paper (see part II), which is currently close to submission to a journal.

I regularly attended the Warsaw arithmetic seminar where participants presented new results in the field. Together with my scientific coordinator and two other participants of

the seminar, prof. Zofia Adamowicz and dr. Konrad Zdanowski, we studied the question of the finite axiomatizability of the arithmetical theory IDelta_0 with a top element and related theories. With the scientific coordinator I tried to make progress on the question of provability of weak versions of the pigeonhole principle in IDelta_0. These investigations have lead to some results, but nothing deserving of publication at this point.

I made some small improvements to a separation result for bounded arithmetic theories R^1_2 and strictR^1_2 which were obtained in my PhD thesis and in part published in Archive for Mathematical Logic. With my scientific coordinator we revisited this topic and made some further observations. I presented the improvements at the conferences listed in part III below and decided that they are not ready for a journal publication yet.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

Title: Some subsystems of constant-depth Frege with parity
Authors: Michal Garlik, Leszek Aleksander Kolodziejczyk
Status: preprint, almost ready for submission
Abstract: A major open problem in propositional proof complexity asks for strong lower bounds on proof size for AC^0[2]-Frege proof systems, that is, proof systems in which lines are constant-depth formulas in the language of NOT, unbounded-fanin AND and OR, and an unbounded-fanin parity connective.
We consider three relatively strong families of subsystems of AC^0[2]-Frege for which exponential lower bounds on proof size are known. In order of increasing strength, these subsystems are: AC^0-Frege with parity axioms in the De Morgan language (that is, with no parity connectives) and the treelike and daglike versions of systems introduced by Krajíček which we call PK^{c}_{d}(\oplus). In a PK^{c}_{d}(\oplus)-proof, lines are cedents in which all formulas have depth at most *d*, parity connectives can only appear as the outermost connectives in formulas, and all but *c* formulas contain no parity connective at all.

We give simple arguments proving that AC^0[2]-Frege is exponentially stronger than daglike PK^{O(1)}_{O(1)}(\oplus), which is exponentially stronger than treelike PK^{O(1)}_{O(1)}(\oplus). On the other hand, we point out that the best known technique for comparing the performance of such systems on De Morgan formulas, due to Impagliazzo and Segerlind, only leads to superpolynomial separations. In fact, we prove that AC^0[2]-Frege with parity axioms and treelike PK^{O(1)}_{O(1)}(\oplus) are quasipolynomially but not polynomially equivalent. This leads us to ask the question whether any of our systems is quasipolynomially equivalent to AC^0[2]-Frege on De Morgan formulas; a positive answer would imply an exponential lower bound for AC^0[2]-Frege.

We also study Itsykson and Sokolov's system Res-lin, in which proof lines are disjunctions of parities of literals. We prove that an extension of treelike Res-lin in which the parity connectives can have arbitrary constant-depth formulas as inputs is quasipolynomially simulated by daglike PK^{O(1)}_{O(1)}(\oplus), and obtain an exponential lower bound for this system.

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

1. FIT 2016 (Forum Informatyki Teoretycznej), Department of Mathematics, Informatics and Mechanics of the University of Warsaw along with the Warsaw Center of Mathematics and Computer Science, 5 – 6 February 2016, Warsaw, Poland
   http://duch.mimuw.edu.pl/~fit2016/
2. Special Semester Program on Complexity Theory, Chebyshev Laboratory, St.Petersburg State University, 14 – 26 May 2016, St.Petersburg, Russia
   http://en.chebyshev.spbu.ru/complexity2016/
3. Journées sur les Arithmétiques Faibles 35, Departamento de Matemática, Faculdade de Ciências da Universidade de Lisboa, 6 – 7 June 2016, Lisboa, Portugal

http://jaf35.campus.ciencias.ulisboa.pt/node/10

4. Logic Colloquium 2016, University of Leeds, 31 July – 6 August 2016, Leeds, United Kingdom
   http://www.lc2016.leeds.ac.uk/

IV – RESEARCH EXCHANGE PROGRAMME (REP)

Dates: 26 June – 3 July 2016
Hosting institute: Universitat Politècnica de Catalunya, Departament de Ciències de la Computació, Barcelona, Spain
Local scientific coordinator: prof. Albert Atserias
I met with prof. Atserias every day and we discussed a possible approach to lower bounds in circuit complexity based on some techniques that have been fruitful in proof complexity. I also met some members of prof. Atserias' research group and together we attended the Logic seminar at the University of Barcelona.