ERCIM "ALAIN BENSOUSSAN" FELLOWSHIP PROGRAMME

Scientific Report

First name / Family name	Kamalakshya Mahatab
Nationality	Indian
Name of the *Host Organisation*	Norwegian University of Science and Technology (NTNU)
First Name / family name of the *Scientific Coordinator*	Kristian Seip
Period of the fellowship	01/10/2016 to 30/09/2017

I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

I have worked on the following two projects during my ERCIM fellowship tenure:

(1) Norms of certain Steinhaus Chaos:

Let z(n) be the Steinhaus random variable, $E_{N,m}:=\{1\leq n:\Omega(n)=m\}$ where $\Omega(n)$ denotes the number of prime factors of n, and $S_{N,m}:=\sum_{n\in E_{N,m}} z(n)$.

Inspired by a paper of A. Bondarenko and K. Seip, we have investigated the following question:

Is there a constant c(k) depending on k and independent of N and m such that $\|S_{N,m}\|_{2k}\ll\|S_{N,m}\|_2$ for $m\leq c(k)\log\log N$?

Bondarenko and Seip have shown that c(2)=1/2 suffice. We have shown that $\|S_{N,m}\|_6\ll\|S_{N,m}\|_2$ when $m\leq(\log\log N)^{\frac{1}{3}}$. We have also conjectured that c(3)=1/4.

(2) Extreme Values of Riemann Zeta:

Granville and Soundararajan proved that

$\zeta(1+it) \geq e^{\gamma}(\log\log t + \log\log\log t - \log\log\log\log t) + O(1)$

for arbitrarily large values of t. We have improved the above bound by removing the factor of log log log t ; we proved that

$\zeta(1+it) \geq e^{\gamma}(\log\log t + \log\log\log t) + O(1)$. This is very close to a conjecture of Granville and Soundararajan. In our proof, we used the resonance method of Soundararajan. Our construction of resonator having an Euler product picks up the large values of ζ from its truncated Euler product on the 1-line.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

[1] C. Aistleitner, K. Mahatab and M. Munsch, Extreme values of the Riemann zeta function on the 1-line, arXiv preprint arXiv:1703.08315, 2017.

Abstract: Let ζ denote the Riemann zeta function. We proved that there are arbitrarily large values of t for which $\zeta(1+it) \geq e^{\gamma}(\log\log t + \log\log\log t) + O(1)$.

[2] K. Mahatab, The 6th Norm of a Steinhaus Chaos, arXiv preprint arXiv:1710.08201

Abstract: We prove that for the Steinhaus Random Variable z(n) and for $S_{N,m} := \sum_{n \leq N, \Omega(n)=m} z(n)$, the 6^{th} and 2^{nd} norms of $S_{N,m}$ are of equal order i.e. $\|S_{N,m}\|_6 \ll \|S_{N,m}\|_2$, when $m \leq (\log\log N)^{\frac{1}{3}}$.

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

(1) Workshop: Analysis Near the Pole, August 7-11 2017, Longyearbyen, Svalbard, Norway.

(2) Conference: Number Theory Week, September 4-8 2017, Faculty of Mathematics and Computer Science of Adam Mickiewicz University, Poznań, Poland.

Seminars:
Influence of Measure on Oscillations of Error Terms, NTNU, Tronheim, 17 October 2016.
Influence of Measure on Oscillations of Error Terms, T U Graz, Graz, 7 March, 2017.
Extreme Values of the Riemann Zeta Function on the 1-line, Analysis Near the Pole, Svalbard, 8 August, 2017.
Influence of Measure on Oscillations of Error Terms, Number Theory Week, Poznan, 5 September 2017.

IV – RESEARCH EXCHANGE PROGRAMME (REP)

Visited Institute of Analysis and Number Theory, T U Graz, Austria from 1st March 2017 to 30th March 2017.
Local Scientific Coordinator: Christoph Aistleitner
Research work:
During this visit I have collaborated with C. Aistleintner and M. Munsch and proved some new results (see [1] in section II) on extreme values of the Riemann zeta function on 1-line.
I also gave talk ``Influence of Measure on Oscillations of Error Terms" during my visit.