



ERCIM "ALAIN BENSOUSSAN"  
FELLOWSHIP PROGRAMME



## Scientific Report

First name / Family name

Dag Nilsson

Nationality

Sweden

Name of the *Host Organisation*

Norwegian University of Science and  
Technology

First Name / family name  
of the *Scientific Coordinator*

Mats Ehrnström

Period of the fellowship

01/09/2018 to 31/12/2019

### I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

The Fractional BBM equation is a model equation used to describe water waves in a certain physical regime. It has been previously shown that the equation is locally well-posed, meaning that local in time there exists a unique solution that depends continuously on the initial data. It has been an open question whether the existence time for the local solution can be increased. I answered this question in the affirmative, extending the existence time using a modified energy method. I benefited greatly from discussions with Mats Ehrnström (NTNU) and Yuexun Wang (NTNU) who have established enhanced existence time for the fractional KdV equation.

I worked on two different Whitham-Boussinesq models both describing waves in a shallow water regime, allowing for bidirectional propagation. In both cases I was interested in existence of solitary wave solutions and we managed to prove existence of such solutions for both models, resulting in two papers. In both papers we use variational techniques to obtain the solutions relying particularly on Lions' concentration compactness theorem. On these papers I collaborated with Yuexun Wang and Evgueni Dinvai (University of Bergen).

I have together with Mats Ehrnström and Mark Groves (Saarland University) been working on proving existence for the Full dispersion KP equation (FDKP) for weak surface tension. The FDKP equation is a model equation for 3d waves involving a nonlocal operator. The proof is variational in nature but differs from the above mentioned works on the Whitham-Boussinesq models. Indeed, a key step in this work is to reduce the functional to a perturbation of a Davey-Stewartson functional. In the end we can show that the FDKP solution possesses solitary wave solutions of Davey-Stewartson type. This work is near completion and we plan to submit a paper in the beginning of 2020.

I have an additional collaboration with Groves where our aim is to prove existence of doubly periodic 3d waves. Hydroelastic waves are waves that propagate on an elastic sheet, an example is waves propagating on a sheet of ice. In this work we use the spatial dynamics approach, meaning that we reformulate the problem as an infinite dimensional dynamical system using a spatial variable as time. The next key step in the proof is to develop a generalized Lyapunov center theorem. We plan to submit a paper in the latter half of 2020.

In the autumn of 2019, I started a collaboration with Shu-Ming Sun (Virginia tech) and Shengfu Deng (Lingnan Normal University) on the existence of double hump solutions to a third order NLS equations (TNLS). The TNLS equation is used in nonlinear optics, and numerical results suggest that this equation does indeed possess double hump solutions, however there is no rigorous proof of this. In this work we intend to remedy this and rigorously prove the existence of double hump solutions, relying on fixed point arguments to construct a solution. This work is still in an early stage and we plan to submit a paper on our results by the end of 2020.

## II – PUBLICATION(S) DURING YOUR FELLOWSHIP

Solitary wave solutions to a class of Whitham-Boussinesq systems, Dag Nilsson, Yuexun Wang. *Z. Angew. Math. Phys.*, 70(3), 2019.

Arxiv: <https://arxiv.org/abs/1810.03405>

Extended lifespan of the fractional BBM equation, Dag Nilsson.

(Pending)

Arxiv: <https://arxiv.org/abs/1902.06336>

Solitary wave solutions of a Whitham-Boussinesq system, Dag Nilsson

(Pending)

Arxiv: <https://arxiv.org/abs/1903.11292>

## III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

Minisymposium 20th anniversary of the Centre for Mathematical Sciences at Lund University, Lund, 10/2019

Norwegian meeting on PDE's, Trondheim, 06/2019

Oberwolfach workshop, Oberwolfach, Germany, 07/2019.

IMACS conference, Athens, Georgia, USA, 04/2019.

## **IV – RESEARCH EXCHANGE PROGRAMME (REP)**