ERCIM "ALAIN BENSOUSSAN" FELLOWSHIP PROGRAMME

Scientific Report

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I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

During this year with the ERCIM fellowship, I have worked on the modelling, and numerical solution, of partial differential equation (PDE) problems related to pricing European and American options considering counterparty risk when stochastic volatility is modeled. Several valuation adjustments are considered, the most common

being the credit value adjustment (CVA), but also debit value adjustment (DVA) and funding value adjustment (FVA) have been computed. In the modelling, the intensity of default from each risky counterparty has been considered constant and we have assumed stochastic volatility, leading to PDE models with two spatial dimensions, the asset price and the volatility. Thus, Cauchy–boundary value PDE problems following Heston model are posed for European options, while complementarity problems govern the pricing of American options. Such models depend on the mark-to-market value, and two common choices for the mark–to–market value, based on the

risk–free and risky derivative values, have been assumed, which lead to linear and nonlinear PDE problems, respectively. For the numerical solution, a method of characteristics jointly with a fixed point iteration and finite elements are used. In the case of American options, an augmented Lagrangian active set method is additionally applied.

The equivalent formulations in terms of expectations have been posed and the exposures associated with the XVA have been numerically computed by means of appropriate Monte Carlo techniques. Moreover, a comparison between the XVA computed considering a Heston model and the XVA computed under the assumptions of Black-Scholes model has been studied. Finally, we have also shown illustrative results of the performance of the models and numerical methods that have been implemented.

In a second part of my research stay at CWI, I have also studied typical problems in finance as pricing European and American options depending on one or two underlying assets. However, instead of addressing these PDE problems applying the most classical techniques used in the literature, based on time and spatial discretization with finite difference or finite element methods, we have proposed some numerical techniques to price European and American options, solving the PDE problem by artificial neural networks. The European option problem can be computed following existing literature for solving PDEs with artificial neural networks. However, the American option problem, modelled as a linear complementarity problem, has not been addressed by artificial neural networks before. Then, novel methodologies have been proposed during the second part of the fellowship, in order to solve classical problems in finance. To compute the solution of such problems, an appropriate loss function has been optimized, moreover we have computed the optimal loss weights in the loss function related to the neural network, to obtain a so-called epsilon-close solution, determining which term of the loss function, the interior or the boundary term, should have more prominent. In addition, the integral terms have been approximated by Monte Carlo techniques, and the neural network weight optimizer used is the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm. Whereas, the most common neural networks in the literature for nonlinear problems, assume a high number of hidden layers and many neurons in each layer, we consider a network composed by 4 hidden layers and 20 neurons per layer. The results obtained have been compared with the analytical solution for the European options, where the classical Black-Scholes formula is known. In the American option case, where an analytical solution is not known, we have used the solution computed by the finite element method as a reference value. This second work, will be sent for publication as a second paper of my work. Highly interesting initial numerical results have been obtained with a novel, nontrivial technique.

Summarizing, during this ERCIM funded year, different financial mathematics problems have been addressed. In the first part of the year, novel financial aspects have been studied, solving the problems using classical numerical methods. However, in the second part, we have solved classical problems in financial mathematics, but they have been solved by novel methodologies in the financial sector, in particular for the American options.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

During this year, one papewr from the PhD period has appeared. Moreover, one paper has been sent for publication (from the first period's work) which is under revision, and we have a second paper in progress.

- Beatriz Salvador and Cornelis W. Oosterlee. Total value adjustment for a stochastic volatility model. A comparison with the lack-Scholes model. Journal of Mathematics and Computation. (Under revision).

  Abstract:

  Since the 2007/2008 financial crisis, the total value adjustment (XVA) should be included when pricing derivatives. In the present paper, the derivative value for European and American options have been priced considering counterparty risk. Whereas European and American options considering counterparty risk have already been priced under Black-Scholes dynamics in [1], here the novel contribution is the introduction of stochastic volatility resulting in a Heston stochastic volatility type partial differential equation to be solved. We derive the partial differential equation modelling the XVA when stochastic volatility is assumed.

  For both European and American options, linear and nonlinear problems have been deduced. In order to obtain a numerical solution, suitable and appropriate boundary conditions have been considered. In addition, a method of characteristic for the time discretization combined with a finite element method in the spatial discretization has been implemented. The expected exposure and potential future exposure are computed to compare the current model with the associated Black--Scholes model.

  References:

  [1] Arregui, I., Salvador, B., Vázquez, C., 2017. PDE models and numerical methods for total value adjustment in European and American options with counterparty risk. Applied Mathematics and Computation 308, 31–53.

  [2] Feng, Q., Oosterlee, C., 2017. Monte Carlo calculation of exposure profiles and greeks for bermudan and barrier options under the Heston Hull-White model. Recent Progress and Modern Challenges in Applied Mathematics, Modeling and Computational Science 79.

  [3] in ’t Hout, K.J., Foulon, S., 2014. ADI finite difference schemes for option pricing in the Heston model with correlation. International Journal of Numerical Analysis and Modeling 7, 1–27.

- Beatriz Salvador, Cornelis W. Oosterlee and Remco van der Meer. Pricing American options by Artificial Neural Networks. (Work in progress).

  Abstract:

  In the last years, artificial neural networks have become very useful solving problems in different fields such as image recognition, data mining, medical diagnosis…Recent works have shown that artificial neural networks can be also applied to solve PDEs. Moreover, this methodology has been extended to solve financial mathematical problems. In this work, we price European and American options, classical problems in finance, following their PDE formulations. In order to solve such PDE problems, instead of using the classical methodologies based on finite element or finite difference methods, we address the problem using artificial neural networks. The novel contribution consists in solving the linear complementarity problems which model American options by ANNs. Moreover, we show the error of the methods, measuring the accuracy with the analytical or numerical solutions computed by finite element method which have been used as a reference solution.

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

During this year with the ERCIM fellowship I have attended to two conferences:
- 3rd International Conference on Computational Finance ICCF 2019. A Coruña, Spain, 8-12th July 2019. Where I presented the work "PDE models for American options with total value adjustment and two stochastic factors". Thanks to this work, I was awarded with the "Journal of Computational Finance Young Researcher Award at ICCF2019. A Coruña, Spain" which was given to the best work presented by a young researcher during the ICCF 2019.
- 2019 Woudschoten Conference Lectures of the forty-fourth Woudschoten Conference. 9-11 October 2019. The Woudschoten Conference Centre in Zeist The Netherlands.

IV – RESEARCH EXCHANGE PROGRAMME (REP)

In June, from 1st to 16th I participated in the research exchange programme (REP) in the research group Mathrisk at INRIA, Research Center of Paris, under supervision of Ludovic Goudenege in Centrale Supélec.

During my stay, I have collaborated with them on the software PREMIA dedicated to quantitative finance, in particular on XVA computation under the Heston dynamics. Mainly, we have solved the problems presented in the first paper, implementing the code with C.