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ERCIM "ALAIN  
BENSOUSSAN" FELLOWSHIP  
PROGRAMME



## Scientific Report

First name / Family name

Jim Tao

Nationality

United States of America

Name of the *Host Organisation*

NTNU

First Name / family name  
of the *Scientific Coordinator*

Franz Luef

Period of the fellowship

01/10/2019 to 31/12/2020

### I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

The main research activity carried out during my ERCIM Alain Bensoussan Fellowship Programme is related to time-frequency analysis and noncommutative tori. Here we will focus on the work completed so far.

In this work, a close relationship between Kähler geometry and Gabor frames has been investigated in the context of index theory results on noncommutative tori. In order to conduct this research we had to develop a novel approach to studying lower frame and interpolation bounds based on Bergman kernel methods.

### II – PUBLICATION(S) DURING YOUR FELLOWSHIP

**Title:** A twisted local index formula for curved noncommutative two tori

**Authors:** Farzad Fathizdeh, Franz Luef, and Jim Tao

**Abstract:** We consider the Dirac operator of a general metric in the canonical conformal class on the noncommutative two torus, twisted by an idempotent (representing the K-theory class of a general noncommutative vector bundle), and derive a local formula

for the Fredholm index of the twisted Dirac operator. Our approach is based on the McKean-Singer index formula, and explicit heat expansion calculations by making use of Connes' pseudodifferential calculus. As a technical tool, a new rearrangement lemma is proved to handle challenges posed by the noncommutativity of the algebra and the presence of an idempotent in the calculations in addition to a conformal factor.

**Status:** A preprint can be found online at <https://arxiv.org/abs/1904.03810>. The paper will be submitted for publication.

### III – ATTENDED SEMINARS, WORKSHOPS, CONFERENCES

#### Workshop

Name: "Equilibrium states in semigroup theory, K-theory and number theory"

Date: 4-6 November 2019

Place: Department of Mathematics, University of Oslo, Norway (I gave a talk)

#### Seminar talks

9 October 2019: NTNU, Operator Algebras Seminar, Trondheim, Norway

29 January 2020: University of Crete, Analysis Seminar, Heraklion, Crete, Greece

6 February 2020: Princeton University, Ergodic Theory and Statistical Mechanics Seminar, Princeton, New Jersey, USA

10 February 2020: Cornell University, Analysis Seminar, Ithaca, New York, USA

21 February 2020: NTNU, Luef's group, Trondheim, Norway

28 February 2020: NTNU, Luef's group, Trondheim, Norway

6 March 2020: NTNU, Luef's group, Trondheim, Norway

9 March 2020: IMPAN, Noncommutative Geometry Seminar, Warsaw, Poland

29 April 2020: St. John's, NYC Noncommutative Geometry Seminar, New York City, New York, USA

### IV – RESEARCH EXCHANGE PROGRAMME (REP)

**Name of the REP organisation:** FORTH-ICS, Crete

**Country:** Greece

**Research group:** Fourier and Functional Analysis

**Local scientific coordinator:** George Makrakis, [g.n.makrakis@iacm.forth.gr](mailto:g.n.makrakis@iacm.forth.gr)

**Dates:** 23-30 January 2020

**Experience:** During my visit at FORTH-ICS, I gave a presentation of my research work on the index theory of vector bundles on the noncommutative torus, and I had the opportunity to exchange ideas with the researchers of the group, George Makrakis and Mihalis Kolountzakis, about potential research collaborations.