ERCIM "ALAIN BENSOUSSAN" FELLOWSHIP PROGRAMME

Scientific Report

First name / Family name	PURNEDU MISHRA
Nationality	India
Name of the *Host Organisation*	University of Warsaw Poland
First Name / family name of the *Scientific Coordinator*	Prof. Dariusz Wrzosek
Period of the fellowship	01/10/2019 to 28/02/2021

I – SCIETIFIC ACTIVITY DURING YOUR FELLOWSHIP

During the ERCIM fellowship period I worked at Faculty of Mathematics, Informatics and Mechanics, University of Warsaw (MIMUW) on a research project that concerns mathematical modelling of interacting species in ecology. Within the project entitled "**The role of chemical signalling and animals' aggregation in predator-prey interactions. Mathematical modelling approach"** which has been carried out with Prof. Dariusz Wrzosek the role of chemical signalling in predator-prey relations was investigated. Two scientific talks were given during the fellowship and I attended few seminars related to bio-mathematics hosted by bio-mathematics and game theory group at MIMUW. I also participated in two international conferences and conducted a week virtual research exchange program with the scientists of INRIA Dracula team, France. The main research activities followed during my ERCIM Fellowship are summarized here:

Under the research project we have published a paper and two papers have been submitted for publications in international journals with high reputation. One another paper related to the project is under early stage of preparation. Traditional models of predator prey interactions assume a uniform in space distribution of individuals which lead to the classical mathematical descriptions expressed in terms of ordinary differential equations or in a more general situation to reaction-diffusion systems in which only diffusive movement of individuals is

taken into account. Recently more attention has been focussed on models which additionally take into account the oriented motion of predators or prey toward or outward gradient of population density or concentration of some chemical. For instance, prey taxis models are concerned in which predators move towards the gradient of prey density or toward the gradient of some chemical secreted by prey. In the published paper (paper 1) we consider a realistic situation which we have modelled mathematically with the following research hypothesis:
- predator exhibits aggregational response with respect to increase in the density of prey,
- predator is capable to detect chemicals secreted by prey,
- a generalist predator has in its diet a prey species under consideration,
- the life span of the predator is significantly longer than that of prey,
- the motility of prey is negligible with respect to predator's motility.

Under the above assumptions on minimalistic predator-prey model studied in the first published work (paper1), it was shown that pattern formation may occur provided chemotactic sensitivity parameter $\chi > 0$ is big enough and for sufficiently high predator density there holds a positive feedback between the predator density and the production of the chemo-attractant released by prey. It was proved that such a positive feedback is due to the interference among predators.

The second work (paper 2) of this project considers the effect of predator evasion mediated by chemical signalling described as chemorepulsion in an extended classical diffusive prey-predator models. We consider the case when the chemical signal is diffusive and plays the role of the alarm signal stimulating evasion as an antipredator response (Model B). We studied two models A and B to verify if classical diffusive predator prey models enriched by terms accounting for chemical signalling can describe the tendency to spatiotemporal separation between prey and predators, by either avoiding areas inhabited by potential predators or using those areas at different times than the predators. Model A accounts additionally for the prey taxis which amounts to the movement of prey towards the gradient of prey density. Existence of global-in-time classical solutions to model A is proved in space dimension $n = 1$ while to model B for any $n > 1$. The most important feature stemming from the stability analysis of the coexistence steady state in Model A and Model B is the destabilizing effect of the repulsive chemotaxis which plays its role even in the case when the prey taxis is concerned. Mathematical analysis reveals that any static bifurcation is precluded and only dynamic bifurcation of Hopf type may exist in the class of models studied in the present paper. Numerical simulations suggest that evasive defence strategy of prey based on chemical signalling may lead to the formation of complex space-time patterns of species distribution. Numerical simulations also suggest that high-dimensional Model A ($n > 1$) may have blow-up of solutions in finite time. Moreover, we have recently started with Dariusz Wrzosek working on a new project related to the present one which is under preparation. The ideas contained in the project motivated yet another article (paper 3) with the Indian colleague. This work concerns the study of a mathematical model in ecology that incorporates the group defence behaviour in fearful prey. Outcome of this study suggests that fear response in predator-prey relation stabilizes the coexistence steady state. However, Turing-alike instability is obtained under certain condition but with the restriction that predator should be much quicker than prey. An important result stemming from the stability analysis confers that not only Turing and Hopf-alike instabilities but also indirect-prey taxis induced instability is responsible for pattern formation if chemo-attractant sensitivity parameter is high enough.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

During the ERCIM training program at MIMUW University of Warsaw I have worked on four research papers in which 1 paper is published, two papers are submitted for publication and 1 paper is currently under preparation. List of papers are:

1. **P. Mishra and D. Wrzosek. The role of indirect prey-taxis and interference among predators in pattern formation.** Published in Mathematical Methods in Applied Sciences Vol. 43 (18) **(2020).**
   [**Abstract**: It is important to find biological factors that may lead to formation of patches in the distribution of species. We build a simple model describing a consumer/predator which, besides random dispersion, searches for food by moving toward the gradient of some chemical released by prey. This mechanism is referred as indirect prey-taxis. The predator's rate of consumption is assumed to drop due to interference among predators when too many of them encounter on some aggregate of prey. The latter is assumed to have a negligible motility with its density governed by an ODE. The interference among consumers is modelled by a modification of the Beddington's de Angelis functional response. Detailed bifurcation analysis and numerical simulations of an auxiliary system indicate that nonconstant steady states imitating patches of species occur in the model provided the predator density exceeds certain threshold and taxis strength is big enough.]
2. **P. Mishra and D. Wrzosek. Repulsive chemotaxis and predator evasion in predator prey models with diffusion and prey taxis**. Submitted in Mathematical Models and Methods in Applied Sciences.
   [Abstract: The role of predator evasion mediated by chemical signalling is studied in a diffusive prey-predator model when prey-taxis is taken into account (Model A) or not (Model B) with taxis strength coefficients χ and ξ respectively. In the kinetic part of the models, it is assumed that the rate of prey consumption includes functional responses of Holling, Bedington-DeAngelis and Crowley-Martin. Existence of global-in-time classical solutions to model A is proved in space dimension $n = 1$ while to model B for any $n \geq 1$. The Crowley-Martin response combined with bounded rate of signal production preclude blow-up of solution in Model A for $n \geq 3$. Local and global stability of a constant coexistence steady state which is stable for purely diffusive model are studied along with mechanism of Hopf bifurcation for Model B when χ is big enough. In Model A it is shown that prey taxis may destabilize the coexistence steady state provided χ and ξ are big enough. Numerical simulation depicts emergence of complex space-time patterns for both models and indicate existence of solutions which blow-up in finite time for $n = 2$.]

3. **P. Mishra and B. Tiwari. Drivers of pattern formation in a predator-prey system with defense in fearful prey**. Submitted in International Journal of Bifurcation and Chaos

   [Abstract: This paper concerns with the spatio-temporal study of a Leslie-Gower predator-prey model with group defense in fearful prey. In order to examine the spatio-temporal dynamics of the proposed model we obtain conditions under which system possess unique global-in-time solutions and determine all the biological feasible states of the system. Local stability of steady states has been investigated with the help of standard linearisation technique and we apply Lyapunov direct method to investigate the global stability of the steady states. We show the occurrence of Hopf-bifurcation and its direction at the vicinity of coexisting equilibrium point for temporal model. We consider random movement in species and establish conditions for the stability of the system in presence of diffusion. We derive conditions for existence of non-constant steady states and Turing instability at coexisting population state of diffusive system. Incorporation indirect prey-taxis with the assumption that the predator moves toward the smell of prey rather than random movement gives rise to taxis-driven instability in predator-prey model. Numerical simulations are intended to demonstrate the role of biological as well as physical drivers on pattern formation that go beyond analytical conclusions.

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

Due to covid-19 restrictions, I attended seminars/conferences in virtual mode only. However, my abstract for a contributory talk was accepted in the international conference MPDEE’20 (University of Leicester UK. 20/04/20-24/04/20) but postponed until May 2021 due to Covid-19. The attended seminars and conferences in virtual model are listed below:

Conferences:

1. Virtual international conference "Dynamics of biological systems: from viruses to populations" organized by Jagiellonian University, Kraków Poland on 23-25 September 2020.
2. Virtual international conference “DYNAMICAL SYSTEMS APPLIED TO BIOLOGY AND NATURAL SCIENCE - DSABNS 2021” organized at BCAM (Spain), INRIA Centre de Reserche Paris (France). University of Sussex (UK).

Seminars:

1. Given a talk on " Mathematical modelling of inhibitory effect in predator-prey systems" at Institute of Applied Mathematics and Mechanics, University of Warsaw Poland on 27 November 2019.
2. Given a talk on "Role of indirect-prey taxis and mutual interference among predator in pattern formation" at Institute of Applied Mathematics and Mechanics, University of Warsaw Poland on 10 June 2020.
3. Given a virtual talk entitled " Nonlinear Differential equations and its application in biology" at Teerthanker Mahaveer University, Moradabad, India on 16 June 2020.
4. Presented my published research paper in a virtual talk during REP at INRIA, France on 23 January 2021.

5. Virtual talk on paper entitled “ "Role of indirect-prey taxis and mutual interference among predator in pattern formation” to research group of Mathematical Modelling of Biomedicine at RUDN University Moscow, Russia.

IV – RESEARCH EXCHANGE PROGRAMME (REP)

Duration: January 25, 2021-January 29, 2021 (1 Week)
Research Group: Inria Dracula team, France
Scientific Contacts: Prof. Vitaly Volpert and Prof. Mostafa Adimy

Summary of REP: As a part of ERCIM fellowship program, I got an opportunity to conduct a research exchange programme with the scientists of DRACULA team at INRIA, France. A one-week (22-25 January, 2021) ERCIM exchange research program held in virtual mode due to COVID-19 restrictions. Previously ERCIM REP was scheduled 27-31 July 2020 at INRIA, France but due to lockdown and covid-19 restrictions in France I was able to travel to France. My research exchange program was hosted with Dracula team scientists Prof. Mostafa Adimy and Prof. Vitaly Volpert. During virtual REP, I shared my research work with the scientists of Dracula team. I discussed on socio-economic models of wealth distribution with Prof. Vitaly Volpert. We had healthy and very informative discussion on wealth distribution models. Prof. Vitaly proposed me to work on wealth distribution model in the near future. Prof. Mostafa discussed with me about stem cell dynamics. We had shared thoughts on mathematical modelling of Wolbachia disease. Prof. Mostafa is opened for collaboration on discussed topics. I learn current research being conducted in the field of biomathematics at Dracula researcher group and exchange the research ideas and my thoughts with other researchers of the team.

Name & Signature of Fellow
(Purnedu Mishra)

Name & Signature of Scientific Coordinator
(Prof. Dariusz Wrzosek)