



ERCIM "ALAIN BENSOUSSAN"
FELLOWSHIP PROGRAMME



Scientific Report

First name / Family name

Alberto Dayan

Nationality

Italy

Name of the *Host Organisation*

NTNU

First Name / family name
of the *Scientific Coordinator*

Kristian Seip & Karl-Mikael Perfekt

Period of the fellowship

01/09/2021 to 31/08/2022

I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

My research activity at NTNU can be summarized by the following three main projects:

- Together with Kristian Seip, Karl-Mikael Perfekt and Andrii Bondarenko (NTNU), I studied the following object

$$F_s(x) := \sum_{n=1}^{\infty} \frac{e(n^2 x)}{n^s},$$

where $e(y) := e^{2\pi i y}$ and s is a complex parameter, whose real part is strictly between 0 and 1. In particular, F_s can be seen both as a Fourier series in x and as a Dirichlet series in s , and both such approaches come with interesting open questions.

By looking at F_s as a Fourier series, we determined the least exponent p so that F_s is in the space L^p to be $\frac{2}{1-\sigma}$, where σ is the real part of s . The motivation for studying such problem comes from a work in progress of Bondarenko and Seip, in which they look for an equivalent re-formulation of the Riemann hypothesis.

On the other hand, by looking at F_s as a Dirichlet series in s , we found its abscissa of convergence. Such abscissa of convergence depends on x , specifically on how well x can be approximated via its continued fractions.

Our research will be soon available as a preprint.

- Together with Adrian Llinares (another ERCIM fellow at NTNU) and Karl-Mikael Perfekt, I investigated the problem of characterizing the trace of a Bloch function on an interpolating sequence. The motivation for this project comes from an early work of Sundberg, who studied the values that a function in BMO can take on an interpolating sequence. Our approach relies on the correspondence, first noticed by Makarov in the 90's, between a Bloch function and a dyadic martingale on the unit circle having bounded jumps. Such probabilistic viewpoint allowed us to turn the problem in a more discrete language, and allowed us to show that, if b is a Bloch function and if $w_n = b(z_n)$, where $(z_n)_{n=1}^{\infty}$ is an interpolating sequence in the unit disc, then there exist positive numbers λ and C and a function g such that, for all z in the unit disc

$$\sum_{n=1}^{\infty} e^{-\log(1-\rho(z,z_n))} \frac{\lambda |g(z) - w_n|^2}{(1 - \rho(z, z_n))} \leq C.$$

Here ρ is the pseudo-hyperbolic distance on the unit disc. Whether such condition is also sufficient, is the object of our current investigation.

- I also started a collaboration with Daniel Seco, from Universidad Carlos III in Madrid and Bartosz Malman, from KTH in Stockholm. We are trying to build a function on the unit disc which is inner for the Dirichlet space while cyclic for the Hardy space. In particular, we are interested in the consequences of an eventual existence of such object for the Brown-Schields conjecture. Our question can be adapted to other settings, as we also ask whether there exists an inner function in the Hardy space which is cyclic for L_A^p , the class of those power series on the unit disc whose coefficients are p -summable.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

The following papers have been accepted while I was an ERCIM fellow:

- Dayan, A.: “Weakly Separated Bessel Systems of Model Spaces”, *Canadian Mathematical Bulletin*
- Dayan, A., Wick, B. and Wu, S.: “Random Interpolating Sequences in the Polydisc and the Unit Ball”, *Computational Methods and Function Theory*.

The following paper has been submitted to a journal for review:

- Dayan, A.: “Interpolating d-tuples of Matrices”

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

While I was an ERCIM fellow, I attended the following conferences:

- *Frontiers of Operator Theory*, CIRM – Luminy, France, November 29th – December 3rd 2021
Talk: “Weakly Separated Bessel Systems of Model Spaces”
- *OTTER Math Meeting* – Online, January 6th -7th 2022
Invited Talk: “Weakly Separated Bessel Systems of Model Spaces”
- *Joint Mathematical Meeting of the American Mathematical Society* - Online, April 6th - 9th 2022
Invited Talk: “Random Interpolating Sequences in the Polydisc”
- *WOTCA 2022* – Faro, Portugal, June 20th - 24th 2022
Invited Talk: “Random Interpolating Sequences in the Polydisc”

I was also invited to give a talk at the Analysis Seminars of the following Departments of Mathematics:

- NTNU – “Random Interpolating Sequences in the Polydisc and the Unit Ball” – October 8th 2021
- Newcastle University – “Random Interpolating Sequences in the Polydisc and the Unit Ball” – Online, March 8th 2022
- University of Saarland – “Dobinski Sets, Function Theory and Sets of Null Capacity” – April 25th 2022

IV – RESEARCH EXCHANGE PROGRAMME (REP)

I did my Research Exchange Programme at the university of Warsaw, in Poland, under the supervision of professor Marcin Moszynski. The week I spent in Warsaw was a great opportunity to expand my area of research, and to meet people who are interested in my early works. In particular, during my PhD years I studied exceptional subsets of the unit interval made of those irrationals that can be approximated exceedingly well by dyadic rationals. I have been interested in studying the size of such sets, both from the viewpoint of Hausdorff measures and Riesz capacities. During my REP I found out that many colleagues in Warsaw are also interested in finding out whether such exceptional sets can support a so called Rajchman measure, that is, a measure whose Fourier coefficients tend to 0 . They also developed some discrete tools, based on dyadic martingales, that fits very well the problem I use to study, which is the reason why I found my Research Exchange Programme particularly fruitful: I got the chance to learn some interesting topics which are very close to one of my areas of interest, and I created the basis for what I hope it will be a fruitful collaboration.