ERCIM "ALAIN BENSOUSSAN"
FELLOWSHIP PROGRAMME

Scientific Report

First name / Family name	Muhammad/Sheraz
Nationality	Pakistani
Name of the *Host Organisation*	Fraunhofer ITWM Kaiserslautern Germany
First Name / family name of the *Scientific Coordinator*	Jörg/ Wenzel
Period of the fellowship	01/01/2022 to 31/12/2022

I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

I worked on various projects during my fellowship at the Department of Financial Mathematics, Fraunhofer ITWM. My research activities mainly focused on the following research projects and scientific publications. Some papers from these projects have been published, and the remaining are planned submissions.

1. In the study of complex systems, statistical relationships such as correlation-based techniques have been extensively used to investigate linear dependence. The most common statistical methods exclusively rely on second-order statistics, such as correlation analysis and Principal Component Analysis (PCA). For example, Granger causality (GC) is based on second-order statistics and focuses on correlation, which constrains its relevance to only linear systems. As a matter of fact, due to the highly increased evidence of non-linear trends in financial time series analysis, we worked on the investigation of volatility dynamics and information-flow between non-linear and highly volatile time series data sets. We propose the employment of Garman and Klass (GK), Parkinson's, Rogers, and Satchell (RS), and Garman and Klass-Yang and Zhang (GK-YZ), Open-High-Low-Close (OHLC) volatility estimators to estimate volatilities. The study applies methods such as mutual information, transfer entropy (TE), effective transfer entropy (ETE), and Rényi transfer entropy (RTE) to quantify the information flow between estimated OHLC-volatilities.

2. Risk assessment represents an important topic in various fields since it allows for designing the optimal strategy for many real-world problems. The fundamental concept of entropy can be used to evaluate the uncertainty degree corresponding to the result of an experiment, phenomenon, or random variable. The uncertainty corresponding to loss random variables in actuarial models can be evaluated also by the entropy of the loss distribution. Frequently in actuarial practice, because of using deductibles and policy limits, the practitioners must deal with transformed data generated by truncation and censoring. The entropic approach enables the assessment of the uncertainty degree

for loss models involving truncated and censored random variables. In this project, we have developed several entropy-based risk models involving truncated and censored loss-random variables. We investigate the effect of various partial insurance schemes, such as truncation and censoring from above, truncation and censoring from below, and inflation using the Tsallis entropy.

3. Nakagami distribution has been introduced and studied by Nakagami as an alternative to Gaussian and Rayleigh distributions. Nakagami distribution is characterised by easy convertibility into some similar distributions. Therefore, resulting distribution characteristics from the conversion process can be transformed by changing Nakagami-parameters. In this project, we estimate the scale parameters of the Nakagami distribution with the method of maximum likelihood. We have introduced the generalization of the Bayesian estimation of the underlying parameter corresponding to the power Nakagami distribution by using different priors such as (i) Jeffreys' prior, (ii) extension Jeffreys' prior, (iii) quasi prior under various symmetric and asymmetric loss functions.

4. Classical portfolio theory by Markowitz (1952), the Black-Scholes model (1973) and other financial literature assume that returns series are distributed normally. Empirical studies suggest that in emerging markets, returns are non-Gaussian with high kurtosis, fat-tails, and non-persistent volatilities. Therefore, it is crucial to comprehend the validity and implications of these assumptions for financial markets on a case-to-case basis. Mandelbrot (1963) proposed a fat-tail distribution model, and consequently, Lévy's stochastic models were proposed in the early 1980s and use to explore different stylized features of financial assets. These models have many appealing properties in financial economics. The contribution of this study is to identify the fat-tailed characteristics of stock and currency returns by employing (i) fat-tailed models, (ii) Gaussian mixtures and (iii) heterogeneous mixtures of underlying distributions to model and compare the performance of each of the models. We test the presence of volatility clustering and leverage effects by using GARCH-type modelling of five different GARCH models such as SGARCH by Bollerslev (1986), GJR-GARCH due to Glosten et al. (1993), EGARCH by Nelson (1991), APARCH by Ding et al. (1993) and CSGARCH by Lee and Engle (1999), each with five conditional distributions. Value-at-Risk (VaR) and discussion on detrended cross-correlation is finally investigated.

5. In the Black & Scholes (BS) model, the stock prices follow the Geometric Brownian Motion process. Therefore, stochastic calculus is vital in this framework. This model provides the framework to extract the future volatility of the underlying asset from the market price of an option. Similarly, the probability distributions of returns for the future prices of an asset can be extracted from the options market. Recently, the employment of information measures has become an alternative tool for solving problems, such as risk-neutral densities (RND) extraction. In this project, we use concepts developed in the entropy pricing theory framework to propose the mathematical construction of several types of RNDs. We present various cases for the construction of an RND based on the weighted entropy, relative weighted entropy, weighted directed divergence and building the minimal weighted divergence martingale measures. The applications of the new theoretical results are discussed in the framework of expected utility-weighted entropy and matching Call and Digital Option prices.

6. Forecasting accuracy is a crucial problem in financial modelling and can be improved based on valid distributional properties of asset returns. The Hidden Markov Model (HMM) is a suitable framework to capture both distributional and temporal properties. The state process of the model evolves as a Markov chain, providing the channel of time dependency. Its distribution is a mixture of several distributions, enabling it to explain the fat tails. HMMs and Hidden Semi-Markov Models (HSMMs) provide flexible, general-purpose models for univariate and multivariate time series, especially for discrete-valued series, categorical series, circular-valued series, and many other types of observations. These models are a special class of mixture models. The hidden Markov/semi-Markov models have many applications for the situations in which there are absorbing or macro-states. In this project, we investigate the problem of forecasting electricity prices from a data science perspective. The electricity spot prices belong to a particular market which is subject to various liberalizations in the European Union and the US. Therefore, such a market is crucial to guarantee the correct price for electricity. In this project I am working with Prof. Dr. Christina Erlwein-Sayer, Dr. Stefanie Grimm, Florian Schirra and Tilman Sayer. We are working in a group for modelling

and pricing energy markets using Hidden Markov Models (HMM), Hidden semi-Markov Models and Long Short-Term Memory (LSTM) networks.

7. Solvency II aims to implement a robust solvency regime for insurance companies that adequately study the actual risk and calculation of the solvency capital requirement (SCR). According to Article 101 (3) of the Directive of the European Parliament and the Council, the SCR shall be calibrated in such a way as to ensure that all quantifiable risks are considered to which an insurance or reinsurance undertaking is exposed. Also, it will cover both the current operations and new operations expected in the following twelve months. It corresponds to the value-at-risk of the basic-own funds of an insurance or reinsurance undertaking at a confidence level of 99.5% over one year. Consequently, the economic balance sheet must be determined based on a market-consistent valuation at today's date (base case) and, using an appropriate approximation method, the expected future cash flows between the insurance company and the policyholders or shareholders (own funds) at the risk horizon (1 year). As part of the Solvency II implementation, it is allowed to use a partial internal model that permits tailor-made capital management. Therefore, insurance companies are ambitious in model development, parameterization, and calibration of the internal model before approval. The aim of the development of the new internal model is to capture all the possible risks faced by the insurance company. Another key factor of the model is the construction of the economic scenarios generator with many economic variables generated by Monte Carlo simulations. In the case of asset dynamics, the economic scenarios generator is used to derive asset returns and valuations. For the desired set of asset classes, the asset model will provide valuations and cash flows framework of a list of securities that an insurance company might like to consider for a portfolio. The asset classes can be classified into (i) fixed income, for example, government bonds or corporate bonds, (ii) derivatives, for example, swaps, currency forwards, and equity derivatives, (iii) fixed income derivatives, and (iv) equity and equity-linked, for example, equity, property, commodities, and hedge funds indices. In this project, I have worked on our inner model that initially consists of (i)interest rate risk, such as changes in term structure /volatility of interest rates, (ii)equity risk (change of market prices of equities),(iii)property risk (change of market price of real estate,(iv)spread risk (changes of credit spread over the risk-free interest rate term structure,(v)currency risk (changes of currency exchange rates), (vi)market risk concentrations and many other factors. In this project I have worked with Prof Dr. Jörg Wenzel, Chair of the project Prof. Dr. Ralf Korn and other team members including Tom Ewen, Simon Schnuerch, Mark-Oliver Wolf and Fatlinda Avdullai.

8. Option pricing is at the center of research in financial mathematics and has received significant attention in the literature. Black and Scholes (BS) (1973) first proposed a solution to the pricing problem known as the Black-Scholes formula. The BS theory establishes that the price of an option primarily depends on the strike, the worth of the underlying asset, volatility, and time to maturity. However, the BS framework only works for European-type call and put options executable at the expiry. BS model fails to provide an analytical solution for American options characterized by a pre-determined fixed strike and Asian options that are even exercisable before the expiry and follows variable (floating) strike. Simulation methods are alternative frameworks to fill the existing void in the literature on pricing Amerasian options. These methods are especially effective under multiple stochastic variables. Niederreiter (1987) first discussed the framework of Monte Carlo methods under different Pseudo-Random- Numbers (PRNs). Further, Tilley (1993) extended the idea to Monte Carlo-based path simulation for option pricing. However, at the early choice of exercise, the underlying option must be priced recursively at each allowed exercise time. In this case, the simulated trajectory at a known time reflects just one possible path, which may provide a biased price estimate. In literature, stochastic simulations and optimizing algorithms offer a flexible solution to price Amerasian options. These models calculate continuation values using a linear combination of polynomial functions. For example, the Least Square Monte Carlo (LSMC) method introduced by Longstaff & Schwartz (2001) is one alternative that employs ordinary least square regression to recursively estimate conditional expectation payoff for continuation at each allowed expiry time. In this project, we are working on characteristic of stochastic state variables in the LSMC framework to improve and compare the performance of our method with existing literature.

9. In quantitative finance and life insurance, various methodologies have been developed to model interest rate structures. In a discrete setting, a binomial like the model of Ho & Lee is well known.

Ho & Lee developed a revolutionary approach for modelling the yield curve movements using a binomial tree. Similarly, continuous-time models of CIR or Hull & White and Vasicek are famous. On the other hand, regime-switching models have been used in financial derivatives, interest rates, and portfolio optimization. For example, Hunt & Devolder used a Shannon minimal-entropy martingale measure for the semi-Markov regime-switching interest rate model in a discrete framework. We introduce the employment of entropy measures that are more sensitive to events that occur many times in a short interval if the power of the probability is positive-large. On the other hand, for negative values, measures are more sensitive to events that occur seldom. In this project, we work to develop the minimal entropy martingale problem for semi-Markov regime switching interest rate models using general entropy measures. A joint representation of extensive and non-extensive entropy measures has been developed. The underlying theoretical properties are investigated and applied to solve the minimal entropy martingale measure problem for deriving risk-neutral densities and interest rate modelling.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

1. Muhammad Sheraz**,** Silvia Dedu, Vasile Preda. Volatility Dynamics of Non-Linear Volatile Time Series and Analysis of Information Flow: Evidence from Cryptocurrency Data. *Entropy* 24, 1410, 2022.
2. Vasile Preda, Silvia Dedu , Iuliana Iatan, Ioana Danila Cernat, Muhammad Sheraz. Tsallis entropy for loss models and survival models involving truncated
and censored random variables, *Entropy* 24, 1654, 2022.
3. Iuliana Iatan, Vasile Preda , Silvia Dedu, Muhammad Sheraz. The Power Nakagami Distribution. Bayesian Method of Estimation using Issues of the Jeffreys' prior, Extension Jeffreys' prior, Quasi prior, under different Loss Functions, (Submitted)
4. Imran Nasir, Muhammad Sheraz, Silvia Dedu. Mixture Models and Modelling Volatility of Returns – A Study on Gaussian and Heterogenous Heavy Tail Mixtures, (accepted)

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

Due to budget constraints, I only visited NTNU for a research exchange program. However, collaborated in two papers presentation at the following conference.

1. The 9th International Conference Economic Scientific Research - Theoretical, Empirical and Practical Approaches ESPERA November 24-25, 2022.

IV – RESEARCH EXCHANGE PROGRAMME (REP)

During my one-week research exchange program (REP) at the Norwegian University of Science and Technology (NTNU), I had the opportunity to exchange ideas with some researchers at the department of Economics. I had several meetings with my host Professor, one meeting with his research collaborator and attended one research seminar on the invitation. My REP was a great and fruitful experience, and I started a joint research project with my host Professor.