ERCIM fellowship Programme

Final scientific report

ERCIM Alain Bensoussan Fellowship Programme

ERCIM
European Research Consortium for Informatics and Mathematics

Fellow	Ganesh Kiran Vaidya
Host Organisation	NTNU, Trondheim, Norway
Scientific coordinator	Helge Holden

I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

During the tenure of ERCIM fellowship at NTNU, I mainly worked on the uniqueness of the entropy solutions of nonlocal conservation laws modeling traffic dynamics, their finite volume approximations, convergence analysis of these approximations and their implementation. Furthermore, I also worked on the rate of convergence of the follow-the-leader models to its macroscopic limit. The accomplished projects are detailed below:

1. **On the accuracy of the finite volume approximations to nonlocal conservation laws:**

In this work, we considered a very general class of these nonlocal conservation laws, namely,

$$\partial_t u + \partial_x(f(u)v(\mu * \beta(u))) = 0 \qquad (t, x) \in Q_T := (0, T) \times \mathbb{R},$$

$$u(0, x) = u_0(x) \qquad x \in \mathbb{R},$$

with f being non-linear and with appropriate regularity on the coefficients v, μ and β. These equations serve as working models for a variety of real-life applications, such as sedimentation, crowd dynamics, vehicular traffic, biological applications in structured population dynamics, supply chain, granular material dynamics and conveyor belt dynamics. In this work, we presented a detailed analysis of the rate of convergence of the existing finite volume approximations of these models and showed it to be 1/2 by proving a novel Kuznetsov-type lemma for nonlocal conservation laws. This work has been published in *Numerische Mathematik (2023)*.

2. **Well-posedness and error estimates for coupled systems of nonlocal conservation laws:**

This work extended the above results to a coupled system of nonlocal hyperbolic conservation laws. These systems can be strongly coupled through the nonlocal coefficient present in the convection term. In this study, we considered a general class of fluxes, where the local part of the flux can be discontinuous at infinitely many points, with possible accumulation points. Here, we established the existence of entropy solutions for such systems with rough local flux, by deriving a uniform BV bound on the numerical approximations. Furthermore, we also proved a general Kuznetsov-type lemma (and hence uniqueness) for such systems with both smooth and rough local fluxes. Finally, we proved the convergence rate of the approximations to the entropy solutions of the system as 1⁄2 and 1⁄3, with homogeneous (in any dimension) and rough local parts (in one dimension), respectively. This work has been published in *IMA Journal of Numerical Analysis (2024)*.

3. **Systems of nonlocal balance laws for dense multilane vehicular traffic:**

In this work, we consider the systems of nonlocal conservation laws with source terms, which can be used to model traffic dynamics in multilane, with the nonlocality present in both convective and source terms. This work establishes the existence of the entropy solution by means of finite volume approximations and the uniqueness through a Kuznetsov-type lemma. Furthermore, the rate of convergence of the numerical schemes to the entropy solution is shown to be 1/2. We study the applicability of the proven theory to a general class of systems of nonlocal balance laws coupled strongly through the convective part and weakly through the source term. This work has been communicated to a journal and is currently *under review*.

4. **Rate of convergence of the follow the leader models to its continuum limit:**

Various models have been proposed in the literature to study the dynamics of traffic flow, both at a microscopic (modeling the interaction of individual vehicles) and macro level (modeling the aggregate behavior of traffic flow, considering traffic flow as a fluid flow). The microscopic models are generally modeled by a system of ODEs, making the systems usually large and numerical algorithms are computationally expensive, whereas the macroscopic models generally follow the laws of fluid dynamics, are modeled by hyperbolic conservation laws, whose literature of theory and numerical algorithms is quite rich. H. Holden and N. H. Risebro (see Netw. Heterog. Media, 13 (3), 409-42, 2018 and Discrete Cont. Dyn. Syst. A, 38 (2), 715-722, 2018) recently proved the solutions of the follow the leader (FTL) model converge to that of LWR model as the number of vehicles goes to infinity. In this context, we aim to prove that the FTL approximations in-fact converge to its continuum limit i.e. the solution of the LWR model at an optimal rate of 1⁄2 in L^1 norm. This is currently *under progress* and is soon expected to be submitted as an article to a journal.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

1. A. Aggarwal, H. Holden and G. Vaidya. On the accuracy of the finite volume approximations to nonlocal conservation laws. *Numer. Math.* (2023).
2. A. Aggarwal, H. Holden and G. Vaidya, Well-posedness and error estimates for coupled systems of nonlocal conservation laws. *IMA J. Numer. Anal.* (2024).
3. S. S. Ghoshal, J. D. Towers and G. Vaidya. BV regularity of the adapted entropy solutions for conservation laws with infinitely many spatial discontinuities. *Netw. Heterog. Media* (2024).
4. A. Aggarwal, H. Holden, G. Vaidya, Systems of nonlocal balance laws for dense multilane vehicular traffic, *Under Review.*
5. A. Aggarwal, G. Vaidya, Convergence of the numerical approximations and well-posedness: Nonlocal conservation laws with rough flux, *Under Review*.
6. H. Holden, G. Vaidya, Rate of convergence of follow-the-leader model to its continuum limit, *Under Preparation.*

III – ATTENDED SEMINARS, WORKSHOPS, CONFERENCES

1. Waves and Nonlinear Phenomena, NTNU, Trondheim, September 28-30, 2022.
2. Workshop on Analysis of PDEs, Karlsruhe Institute of Technology, Germany, March 27-31, 2023.
3. Workshop on nonlocal and nonlinear PDEs, NTNU, Trondheim, May 24-26, 2023.

IV – RESEARCH EXCHANGE PROGRAMME (REP)

Local coordinator: Prof. Laura Spinolo
Host Organization: IMATI-CNR, Pavia
Duration: Jan 16-26, 2024

During this visit, we discussed various aspects of systems of local and nonlocal conservation laws. I was also given the opportunity to present my works on nonlocal conservation laws.