Fellow	Pranendu Darbar

Host Organisation	Department of Mathematical Sciences, NTNU, Trondheim

Scientific coordinator	Kristian Seip

I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

The projects undertaken during the fellow's tenure at NTNU can be encapsulated as follows:

1. In a joint work with Andriy Bondarenko (NTNU), Markus V. Hagen (NTNU), Winston Heap (NTNU) and Kristian Seip (NTNU), the fellow exhibit large values of the Dedekind zeta function of a cyclotomic field on the critical line. This implies a dichotomy whereby one either has improved lower bounds for the maximum of the Riemann zeta function, or large values of Dirichlet L-functions on the level of the Bondarenko–Seip bound. This work has been published in Bulletin of London Mathematical Society.

2. In a joint work with Gopal Maiti (University of Marseille, France), the fellow proved an Ω-result for the quadratic Dirichlet L-functions $L(1/2,\chi_P)$ over irreducible polynomials P associated with the hyperelliptic curve of genus g over a fixed finite field F_q in the large genus limit. In particular, we showed that for any $\epsilon \in (0,1/2)$,

$$\max_{P \in \mathscr{P}_{2g+1}} |L(1/2,\chi_P)| \gg \exp\left(\left(\sqrt{(1/2-\epsilon)\ln q} + o(1)\right)\sqrt{\frac{g \ln_2 g}{\ln g}}\right),$$

where $\mathscr{P}_{2g+1}$ is the set of all monic irreducible polynomial of degree 2g+1. This matches with the order of magnitude of the Bondarenko–Seip bound. The manuscript has been accepted for publication in Proceedings of American Mathematical Society.

3. In a joint work with Chantal David (Concordia University, Canada), Matilde Lalin (University of Montreal, Canada) and Allysa Lumley (York University, Canada), the fellow investigated the distribution of values of cubic Dirichlet L-functions at s = 1. Following ideas of Granville and Soundararajan for quadratic L-functions, they model the distribution of $L(1, \chi)$ by the distribution of random Euler products $L(1, X)$ for certain family of random variables $X(p)$ attached to each prime. They obtain a description of the proportion of $|L(1, \chi)|$ that are larger or that are smaller than a given bound, and yield more light into the Littlewood bounds. Unlike the quadratic case, there is an asymmetry between lower and upper bounds for the cubic case, and small values are less probable than large values. This work has been submitted for publication.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

The fellow published the following articles during the fellowship:

1. *Large values of quadratic Dirichlet L-functions over monic irreducible polynomials in $F_q[t]$, To appear in Proceedings of American Mathematical Society, 2024. Authors: Darbar, Pranendu; Maiti, Gopal*

2. *A dichotomy for extreme values of zeta and Dirichlet L-functions, Bulletin of the London Mathematical Society, 55 (2023), 2963-2975. Authors: Bondarenko, Andriy; Darbar, Pranendu; Hagen, Markus; Heap, Winston; Seip, Kristian*

3. *Correlation of multiplicative functions over $F_q[x]$: a pretentious approach, Mathematika, 70 (2023). Authors: Darbar, Pranendu; Mukhopadhyay, Anirban*

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

The fellow has participated in the following conferences/workshops/research visits:

1. Excursions in Analysis, Trondheim (Norway), June 2022.
2. ELAZ 2022, Poznan, Poland, August 2022.
3. Conference on Analytic Number Theory, ISI Kolkata, India, March 2023.
4. ArStAFANT Workshop, EPF Lousanne, Switzerland, June 2023.
5. JA2023 : 32èmes Journées Arithmétiques, Nancy, France, July 2023.
6. Mittag-Leffler Institute, Stockholm, Sweden, 4 - 13 February 2024.
7. VTT, Espoo, Finland, 27 February to 4 March 2024; Host- Jorma Kilpi
8. University of Turku, Turku, Finland, 5 - 7 March 2024; Host- Kaisa Matomäki .

The fellow has been given the following seminar/ conference talks:

1. Mesoscopic fluctuation of zeros of L-functions over function fields via Selberg's theorem, Turku Number Theory Seminar, Turku, Finland, March 2024.
2. Mesoscopic fluctuation of zeros of L-functions over function fields via Selberg's theorem, ISI Kolkata, India, January 2024.
3. Extreme values of the Dirichlet L-functions on the critical line, IMSc, Chennai, India, January 2024.
4. Asymmetric distributions of the extreme values for cubic L-functions on the 1-line, Centre Bernoulli, EPF Lausanne, June 2023.
5. Multiplicative functions in short intervals, NISER Bhubaneswar, India, March 2023.
6. Conference on Analytic Number Theory, ISI Kolkata, March 2023.
7. PIMS CRG Weekly seminar series (online), Canada, September 2022.
8. Selberg's central limit theorem over function fields, ELAZ 2022, Poland, August 2022.

IV – RESEARCH EXCHANGE PROGRAMME (REP)

The fellow visited VTT, located in Helsinki (Finland), from 27 February to 7 March 2024. Hosted by Jorma Kilpi, the stay provided an excellent opportunity for the exchange of ideas. During the visit, the visitor delivered an engaging presentation on their research titled "The gap between primes and product of two primes" shedding light on the intersection of number theory with the practical realms of cryptography and quantum computing. This experience served as a catalyst, inspiring future collaboration with Dr. Kilpi.