



Fellow	<Malte / Gerhold >
Host Organisation	<NTNU>
Scientific coordinator	<Franz / Luef >

I - SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

Together with Franz Luef, I explored the possibility of applying recent results from dilation theory to problems in Gabor analysis. This work resulted in a yet unpublished draft (→ II.C.1), where we prove that the frame bounds of a Gabor frame depend $1/2$ -Hölder continuously on the lattice for Gabor atoms from the modulation space M^1_2 . In fact, the methods apply much more generally, and we find that the Heisenberg modules underlying the the duality in Gabor analysis form a continuous field of Hilbert C^* -modules.

I continued my previous research cooperations. I visited Orr Shalit at the Technion twice; 2022 May 15 – 26: research on dilations of unitary groups (→ II.A.1), 2023 February 27 - March 09: research on almost q -commuting unitaries (→ II.C.2). Instead of extending our work to operator systems generated by block unitaries, these other directions seemed more fruitful with possible applications in time-frequency analysis in mind, which is why I slightly shifted the focus compared to the submitted research project. I worked online with I. Baraquin, U. Franz, A. Kula, M. Tobolski on cohomology of quantum groups (→ II.C.3), with J. Diehl and N. Gilliers (→ II.A.3), with T. Hasebe and M. Ulrich (→ II.A.2), and with P. Varšo (→ II.B.1). I also revised two earlier articles (→ II.A.4, II.B.2).

I aquired funding (10000 NOK from *Trond Mohn Foundation via Pure Math in Norway*) to invite Michael Skeide to NTNU. While the visit was originally planned for summer 2022, it had to be postponed to early 2023 (January 17 – February 02), caused by the strike of Scandinavian airlines. During his visit I organized together with Eduard Ortega a one-day seminar (funded with 50000 NOK from *Trond Mohn Foundation via Pure Math in Norway*). Besides Michael Skeide, Are Austad from University of southern Denmark and several guests from University of Oslo participated (→ III.C.1). From January 30 to February 1, we organized a short lecture series (Skeide: " C^* -correspondences and Morita equivalence of Hilbert modules", Luef: "Heisenberg modules and Gabor analysis", Gerhold "Dilations and noncommutative tori") to which also PhD students were invited.



II - PUBLICATION(S) DURING YOUR FELLOWSHIP

A. Journal articles (accepted)

1. Gerhold, M.; Shalit, O.: Bounded perturbations of the Heisenberg commutation relation via dilation theory. To appear in Proceedings of the AMS, [doi:10.1090/proc/16456](https://doi.org/10.1090/proc/16456)
2. Gerhold, M.; Hasebe, T.; Ulrich, M.: Towards a Classification of Multi-Faced Independence: A Representation-Theoretic Approach. To appear in Journal of Functional Analysis, [doi:10.1016/j.jfa.2023.109907](https://doi.org/10.1016/j.jfa.2023.109907)
3. Diehl, J.; Gerhold, M.; Gilliers, N.: Shuffle Algebras and Non-Commutative Probability for Pairs of Faces. SIGMA Symmetry Integrability Geom. Methods Appl. 19 (2023), Paper No. 006, 50 pp., [doi:10.3842/SIGMA.2023.006](https://doi.org/10.3842/SIGMA.2023.006)
4. Gerhold M.; Lachs, S.; Schürmann, M.: Categorical independence and Lévy processes. SIGMA Symmetry Integrability Geom. Methods Appl. 18 (2022), Paper No. 075, 27 pp., [doi:10.3842/SIGMA.2022.075](https://doi.org/10.3842/SIGMA.2022.075)

B. Preprint (pending)

1. Gerhold, M.; Varšo, P.: Towards a classification of multi-faced independences: a combinatorial approach. Preprint, [arXiv:2301.01816](https://arxiv.org/abs/2301.01816)
2. Gerhold, M.: Schoenberg correspondence for multifaced independence. Preprint, [arXiv:2104.02985v3](https://arxiv.org/abs/2104.02985v3)

C. In preparation (unpublished)

1. Gerhold, M.; Luef F.: Dilations and Gabor Analysis (6 pages draft)
2. Gerhold, M.; Shalit, O.: Dilations of almost Θ -commuting unitaries (10 pages draft)
3. Baraquin, I.; Franz, U.; Gerhold, M.; Kula, A.; Tobolski, M.: A free resolution for the free unitary quantum group (22 pages draft)



III - ATTENDED SEMINARS, WORKSHOPS, CONFERENCES

A. Seminar talks

1. Universal products of representations and multi-faced independences, invited by Orr Shalit, 2023 March 8, Haifa, Israel
2. Bounded perturbation of the Heisenberg commutation relation via dilation theory, local analysis seminar, 2022 September 5, Trondheim, Norway
3. Continuity of frame operators in Gabor analysis via dilations of q -commuting unitaries, invited by Orr Shalit, 2022 May 25, Haifa, Israel
4. Dilation distance and noncommutative tori, local analysis seminar, 2022 April 27, Trondheim, Norway

B. Conference participation

1. **2022 RIMS Mathematical aspects of quantum fields and related topics**, 2023 January 16 - 18, Research Institute for Mathematical Sciences RIMS (Kyoto University)
talk: Classification of two-faced independences
2. **43rd International Conference on Quantum Probability and Related Topics**, 2023 January 8 - 14, Utop Marina, Yeosu, Korea
talk: Classification of two-faced independences
3. **Mini workshop on quantum groups, graphs and symmetries via representation theory**, 2022 November 15 - 17, Saarbrücken, Germany
talk: Classification of two-faced independences
(during the intermission of the fellowship)
4. **Noncommutative Harmonic Analysis and Quantum Groups**, 2022 September 11 - 16, Będlewo, Poland,
talk: Bounded perturbations of the Heisenberg commutation relation via dilation theory
5. **Nasjonalt Matematikermøte** (National meeting of Mathematicians in Norway), September 1 - 2, Tromsø, Norway

C. Conference organization

1. **Dynamics in the Noncommutative World** (TMS Joint Day Seminar), NTNU Trondheim, 2023 January 26; Organizers: Malte Gerhold and Eduard Ortega
Acquired funding: 50000 NOK from *Trond Mohn Foundation* via *Pure Math in Norway*



IV - RESEARCH EXCHANGE PROGRAMME (REP)

2022 September 19 - 23, Warsaw University, Poland, local scientific coordinator: Piotr Soltan

During my one week stay, I discussed with Piotr Soltan (Warsaw U.), Paweł Kasprzak (Warsaw U.), Adam Skalski (IMPAN Warsaw), and Mateusz Wasilewski (IMPAN Warsaw) about the possibility of constructing noncommutative tori with parameter in the unit disk. We identified two possible approaches to the problem. However, the problem remains open and needs further investigation.