Fellow	Trung Hieu **VU**
Host Organisation	Norwegian University of Science and Technology
Scientific coordinator	Elisabeth Anna Sophia **Köbis**

I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

1. **On structure of solutions in vector optimization.** My colleagues at Norwegian University of Science and Technology and I proposed criteria for unboundedness of the images of set-valued mappings having closed graphs in Euclidean spaces. We focused on mappings whose domains are non-closed or whose values are connected.

These criteria allow us to see structural properties of solutions in vector optimization, where solution sets can be considered as the images of solution mappings associated to specific scalarization methods. In particular, we have proved that if the domain of a certain solution mapping is non-closed, then the weak Pareto solution set is unbounded. Furthermore, for a quasi-convex problem, we demonstrated two criteria to ensure that if the weak Pareto solution set is disconnected then each connected component is unbounded.

2. **On computing local minimizers in polynomial optimization.** My colleague at the University of Tokyo and I focused on computing local minimizers of a multivariate polynomial optimization problem under genericity conditions.

By using a technique in computer algebra and the second-order optimality condition, we provided a univariate representation for the set of local minimizers. In particular, for the unconstrained problem with n variables, the coordinates of all local minimizers can be represented by the values of n univariate polynomials at real roots of a system including a univariate polynomial equation and a univariate polynomial matrix inequality. We also developed the technique for constrained problems having equality/inequality constraints. Based on the above technique, we designed symbolic algorithms to enumerate the local minimizers and provide some experimental examples based on hybrid symbolic-numerical computations. For the case that the genericity conditions fail, we proposed a perturbation technique to compute approximately a global minimizer provided that the constraint set is compact.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

[1]. Vu Trung Hieu, Elisabeth Anna Sophia Köbis, Markus Arthur Köbis, Paul Hugo Schmölling: *Unboundedness of the images of set-valued mappings having closed graphs: Application to vector optimization*; https://arxiv.org/abs/2312.14783; submitted

[2]. Vu Trung Hieu, Akiko Takeda: *Computing local minimizers in polynomial optimization under genericity conditions*; https://arxiv.org/abs/2311.0083; submitted

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

April 26-28, 2023: Norwegian Workshop on Mathematical Optimization, Nonlinear and Variational Analysis, NTNU, Trondheim, Norway (exhibition of the poster *Unboundedness of solution sets in vector optimization*)

September 1, 2023: *Some topics of polynomial optimization*, Continuous Optimization Team Seminar, The University of Tokyo and RIKEN, Japan

December 12, 2023: *Computing local minimizers in polynomial optimization*, Mathematics & Decision Conference, Université Mohammed VI Polytechnique, Morocco

March 28, 2024: *A hybrid symbolic-numerical algorithm to enumerate local minimizers in polynomial optimization*, Workshop on Scientific Computing and Applications, Vietnam Institute for Advanced Study in Mathematics, Hanoi, Vietnam

April 2, 2024: *Computing local minimizers in polynomial optimization*, 22nd Workshop on Optimization and Scientific Computing, Ba Vi, Vietnam

June 4, 2024: *Exact certificates of non-negativity, computation and structure of solutions in polynomial optimization*, Applied and Computational Algebra Team Seminar, The University of Tromsø, Norway

IV – RESEARCH EXCHANGE PROGRAMME (REP)

Due to time restrictions, the research exchange programme with an another ERCIM institution could not be completed. However, a stay at the University of Tokyo for a collaboration with Prof. Akiko Takeda was made possible. This fruitful collaboration resulted in one publication and a postdoc-position after the completion of the ERCIM scholarship.