

ERCIM fellowship Programme Final scientific report



Fellow

Swati **Yadav**

Host Organisation

NTNU, Trondheim

Scientific coordinator

Mats **Ehrnström**



I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

During the fellowship period I mainly focused on the research projects outlined below:

1. Direct construction of solitary waves for a fractional Korteweg-de Vries equation with an inhomogeneous symbol.

Together with Jun Xue, I worked on the direct construction of solitary waves for a fractional Korteweg-de Vries equation

$$u_t + (\Lambda^{-s}u + u^2)_x = 0, \quad s > 0$$

Where $u(t, x)$ is the deflection of a water fluid surface from the rest position at time $t \geq 0$ and position $x \in \mathbb{R}$ and Λ^{-s} is Bessel potential operator given by

$$\Lambda^{-s} : f \rightarrow \mathcal{F}^{-1}(\langle \xi \rangle^{-s} \hat{f}(\xi))$$

with inhomogeneous symbol of order $-s$

$$\langle \xi \rangle^{-s} := (1 + \xi^2)^{-\frac{s}{2}}.$$

The method involves parameterizing the known periodic solution curves in terms of the relative wave height. Using a priori estimates for these solutions, we establish that the periodic waves converge locally uniformly to waves with negative asymptotic tails, which are subsequently transformed into the desired solution branch. This resulting branch originates from the trivial solution and evolves towards the highest wave through a unique trajectory in the wave speed-wave height phase space. The behaviour of the highest wave near its crest is dependent on the parameter s and reflects the characteristics of the corresponding periodic waves. Moreover, the constructed solitary waves are symmetric and decay exponentially.

The results of this project are expected to be submitted to a journal as an article soon.

2. Construction of Data-driven PDEs in connection to water waves.

This is a joint project led by Mats Ehrnström, where we are trying to build dispersive PDEs from data. The theoretical framework, which ensures the retention of classical and established equations under a relatively general ansatz, is being developed by Ehrnström and Katerina Nik. My role in this joint analytical-numerical project is to implement numerical methods to validate the theoretical results. Even though the project is taking more time than we first thought, we're still determined to finish it. The project steps are as follows:

- **Implementing the inverse linear problem** for equations of the form $u_t + m(D)u_x = 0$, where $m(\xi) = \sum_j c_j m_j(\xi)$ is a weighted sum, being a partition of unity when $c_j = 1$ for all j .
This task is in its final stage.
- **Implementing the forward solution** of the same problem for specified initial conditions.



- If possible, **deriving error estimates** for the error between the real and found parameters, and the real data and induced solution u .

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

- With Jun Xue, A direct construction of solitary waves for a fractional Korteweg-de Vries equation with an inhomogeneous symbol. (ready for submission)
- With Mats Ehrnström and Katerina Nik, Recapturing water wave model equations from real and generated data. (In preparation)

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

Workshops and Conferences

3rd Norwegian meeting on PDEs, University of Oslo, 2024 June 5-7.

Poster: A direct construction of solitary waves for a fractional KdV equation with an inhomogeneous symbol.

EWM-EMS Summer School: Water Waves and Nonlinear Dispersive Equations, Mittag-Leffler institute, Sweden, 2024 May 27-31.

Dispersive and water waves seminar, Bergen, 2024 April 27-18.

Talk: A Direct Construction of Solitary Waves for a Fractional Korteweg-De Vries Equation with an Inhomogeneous Symbol.

Workshop on PDEs, Spatio-Temporal Statistics, and Data-Driven Methods in Neuroscience and Fluid Mechanics, Trondheim, Norway, 2024 March 12-14.

21ST IMACS WORLD CONGRESS, Rome, 2023 September 11-15.

Talk: Construction of Solitary Waves for Fractional K-dV Equation with Inhomogeneous Symbol.

10th ICIAM 2023 (online), Tokyo, 2023 August 20-25.

Talk: Numerical Approximation of Burgers Equation with Non-Singular Time-Fractional Derivatives.

Workshop on nonlocal and nonlinear PDEs, NTNU, Norway, 2023 May 24-26.

The Abel symposium, Norway, 2023 June 13-16.

Workshop on Analysis of PDEs, Karlsruhe Institute of Technology, Germany, 2023 March 27-31



IV – RESEARCH EXCHANGE PROGRAMME (REP)

Host organization: Applicazioni del Calcolo del CNR, Italy

Dates: 2023 September 18-22

Local coordinator: Roberto Natalini

The visit allowed me to engage in engaging discussions with Roberto Natalini and Gabriella Bretti about the potential applications of non-local operators in modelling real-world phenomena, such as the flow of water in porous media. I also had the opportunity to present my research work. This visit opened the door for future collaboration with Gabriella Bretti.