



ABCDE



## Scientific Report

First name / Family name

Ishtiaq Ali

Nationality

Pakistani

Name of the *Host Organisation*

PLERCIM

First Name / family name  
of the *Scientific Coordinator*

Urszula Forys

Period of the fellowship

01/07/2012 to 30/06/2013



## I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

I have been affiliated with the Institute of Applied Mathematics and Mechanics of the Faculty of Mathematics, Informatics and Mechanics, at University of Warsaw. Since 2007 the faculty with at most 250 members and Ph.D students is a full member of the European Research Consortium for Informatics and Mathematics (ERCIM). I joined the group of Mathematical Biology, which involves mainly in mathematical description of various biological processes and the analysis of mathematical models (ODEs, PDEs, delay equations, integro-differential equations, Markov processes, difference equations) arising in mathematical biology. Before joining the group, my research interest was concerned with the numerical solutions of delay differential equation, where I mainly focused on the convergence analysis of the proposed method. In the first half of my fellowship, I focused on the numerical solutions of integro-delay differential equations with weakly singular kernels. The detail convergence analysis was provided in the infinity norm for the said equation with proportional delays. In the second half of my fellowship I mainly focused on the spectral postprocessing technique for one and two-stage model of carcinogenesis mutations with time delay and diffusion, which arise from the mathematical modeling of the tumor growth. The main purpose was to construct an efficient and stable discretization scheme which allows much accuracy than those of a standard approaches. The results obtained from the simulations of one-stage model were compared with previously published ones and we found it with good agreement with it, when both the delays are equal to zero, or positive.

## II – PUBLICATION(S) DURING YOUR FELLOWSHIP

Title: Jacobi-spectral methods for integro-delay differential equations with weakly singular kernels

Status: Prepared

Authors: Ishtiaq Ali

**Abstract:** We present a numerical solution to the integro-delay differential equation with weakly singular kernel with the delay function  $\theta(t)$  vanishing at the initial point of the given interval  $[0, T]$ , ( $\theta(t) = qt$ ,  $0 < q < 1$ ). In order to fully use the Jacobi orthogonal polynomial theory, we use some function and variable transformation to change the integro-delay differential equation into new equation defined on the standard interval  $[-1, 1]$ . The solution of the new equation possesses a better regularity property. A Gauss-Jacobi quadrature formula is used to evaluate the integral term. The spectral rate of convergence is provided in infinity norm under the assumption that the solution of the given equation is sufficiently smooth. For the validation of theoretical exponential rate of convergence of our method, we provide some numerical examples.



Title: Spectral postprocessing technique for one and two-stage model of carcinogenesis mutations with time delay and diffusion

Status: In preparation

Authors: Ishtiaq Ali

**Abstract:** In this paper we focused on the numerical solutions of carcinogenesis mutations models that are based on reaction-diffusion systems and Lotka-Volterra food chains. We consider the case with one and two stages of mutations with zero-flux boundary conditions. The main purpose is to construct an efficient and stable discretization scheme which allows much accuracy than those of a standard approach. To this end, we use the spectral method to postprocess numerical solutions for the proposed model obtained by some classical methods for differential equations. We simulate the one and two-stage carcinogenesis mutations model and compared the results with previously published ones.

### III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

- Seminar in Biomathematics and the Game Theory, a weekly scientific meeting where department members present their own research and results in turn, where I also gave one seminar.
- The European Research Consortium for Informatics and Mathematics Fall meeting, INRIA, Sophia Antipolis, FRANCE, October 24-25, 2012.
- SIAM, Conference on Applications of Dynamical Systems, May 19-23, Snowbird, Utah, USA, attended with a talk.

### IV – RESEARCH EXCHANGE PROGRAMME (REP)

In the framework of the Exchange Research Programme I have visited the following two ERCIM institutes

NTNU:

Contact Person: Prof. Espen R Jakobsen

Department of Mathematical Sciences

Norwegian University of Science and Technology,

N-7491 TRONDHEIM.

Office: (+47) 73 59 35 12

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Email: [erj@math.ntnu.no](mailto:erj@math.ntnu.no)

URL: <http://www.math.ntnu.no/~erj>



Duration: November 26-30, 2012

During this visit I gave a seminar at the group of differential equation and numerical analysis at NTUN. I had a numerous discussions with Prof. Jakobsen on the research problems of our mutual interest and agreed to work on some problems in future.

INRIA:

Contact Person: Prof. Mostafa Adimy,

INRIA Grenoble - Rhône-Alpes

Inovallée 655 avenue de l'Europe Montbonnot

38 334 Saint Ismier Cedex France.

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URL: <http://www.inria.fr/equipes/dracula>

E-mail: [mostafa.adimy@inria.fr](mailto:mostafa.adimy@inria.fr)

Duration: April 14-20, 2013

During this visit Prof. Mostafa introduced me to all the members of his group. I gave a seminar to the group of team DRACULA and then worked on one my paper. Prof. Mostafa take a keen interest in the work of my group at PLERCIM and was eager to start collaboration with them in future.

### **Other Activities:**

During the fellowship I regularly reviewed research articles for some reputed journals, for example, Applied Mathematical Modeling, Journal of the Franklin Institute, BIT Numeric, Mathematical and Computer Modeling, etc.