ERCIM Alain Bensoussan Fellowship Programme

ABCDE

MARIE CURIE ACTIONS

Scientific Report

First name / Family name	Anargyros Katsampekis
Nationality	Greek
Name of the *Host Organisation*	CWI
First Name / family name of the *Scientific Coordinator*	Monique Laurent
Period of the fellowship	01/10/2012 to 30/09/2013

I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

My research activity during the ERCIM fellowship at CWI was concentrated on two directions:
1. Describing the structure of the universal Gröbner basis of an ideal generated by diagonal 2-minors and
2. Computing the exact value of the binomial arithmetical rank of a lattice ideal.
The above mentioned activities are briefly analysed below.

1. Describing the structure of the universal Gröbner basis of an ideal generated by diagonal 2-minors.

My approach for this problem involved toric ideals, a well-studied class of binomial ideals. Since any ideal P_G generated by diagonal 2-minors is also a toric ideal, the first step was to find a vector configuration A_G such that P_G equals the toric ideal associated to A_G. Next, I used this configuration to prove that if G is bipartite then, P_G is a unimodular toric ideal. As a consequence, for every bipartite graph G the initial ideal of P_G is generated by squarefree monomials. Moreover, the universal Gröbner basis of P_G is determined by the circuits of the toric ideal associated to A_G, when G is a bipartite graph. To find the circuits of the above toric ideal, I studied the problem of when the ideal P_G can be realised as the toric ideal associated to a finite simple graph. I proved that this is exactly the case when every connected component of G has at most one cycle. Furthermore, I showed that:
(a) If G is a tree, then the universal Gröbner basis consists of all binomials which correspond to even cycles of the prism of G.
(b) If G is a connected bipartite graph with exactly one cycle, then the universal Gröbner basis consists of all binomials which correspond to even cycles of an appropriate graph H.
Finally, I explicitly calculated the number of elements and the maximum degree in the universal Gröbner basis of P_G, when G is either a star graph or a path graph.

2. Computing the exact value of the binomial arithmetical rank of a lattice ideal.

Lattice ideals arise naturally in problems from diverse areas of mathematics. This class of ideals includes also the equicut ideals and the ideals generated by diagonal 2-minors. A basic problem in Commutative Algebra is to determine the binomial arithmetical rank of a lattice ideal. To deal with this problem I considered the indispensable monomials of a lattice ideal I_L and introduced the simplicial complex Γ_L. Furthermore, I used matchings in simplicial complexes to provide a lower bound for the binomial arithmetical rank of a lattice ideal. This lower bound was computed in two cases:
(a) I_L is a toric ideal associated to a number of graphs, including the wheel graph and a weakly chordal graph.
(b) I_L is a determinantal ideal generated by the 2x2 minors of a certain matrix of indeterminates.
In both cases the binomial arithmetical rank equals the minimal number of generators of

the lattice ideal.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

1. A. Katsabekis (Katsampekis). Toric ideals and diagonal 2-minors, submitted for publication to a scientific journal (pending), available at http://front.math.ucdavis.edu/1308.4308.

Abstract. Let G be a simple graph on the vertex set $\{1,\ldots,n\}$. An algebraic object attached to G is the ideal P_G generated by diagonal 2-minors of an $n \times n$ matrix of variables. In this paper we first provide some general results concerning the ideal P_G. It is also proved that if G is bipartite, then every initial ideal of P_G is generated by squarefree monomials. Furthermore, we completely characterize all graphs G for which P_G is the toric ideal associated to a finite simple graph. As a byproduct we obtain classes of toric ideals associated to non-bipartite graphs which have quadratic Gröbner bases. Finally, we provide information in certain cases about the universal Gröbner basis of P_G.

2. A. Katsabekis (Katsampekis). On the binomial arithmetical rank of lattice ideals, submitted for publication to a scientific journal (pending), available at http://front.math.ucdavis.edu/1304.6607.

Abstract. To any lattice $L \subset Z^m$ one can associate the lattice ideal $I_L \subset K[x_1,\ldots,x_m]$. This paper concerns the study of the relation between the binomial arithmetical rank and the minimal number of generators of I_L. We provide lower bounds for the binomial arithmetical rank and the A-homogeneous arithmetical rank of I_L. Furthermore, in certain cases we show that the binomial arithmetical rank equals the minimal number of generators of I_L. Finally we consider a class of determinantal lattice ideals and study some algebraic properties of them.

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

1. **Attended** *ERCIM ABCDE Seminar* at INRIA, Sophia Antipolis, France, 24/10/2012-25/10/2012.
2. **Attended** *Intercity Number Theory Seminar* at University of Eindhoven, Eindhoven, The Netherlands, 02/11/2012.
3. **Talk** *Binomial arithmetical rank of toric ideals associated to graphs*. Working group Algebra and Combinatorics, CWI, Amsterdam, The Netherlands, 18/01/2013**.**
4. **Talk** *Circuits of vector configurations and the binomial arithmetical rank of toric ideals of graphs*. Algebraic Geometry seminar at Polytechnic University of Catalonia, Barcelona, Spain, 22/02/2013**.**
5. **Attended** *4th SDP days* at CWI, Amsterdam, 21/03/2013-22/03/2013.

6. **Talk** *Binomial arithmetical rank of toric ideals associated with graphs*. Algebraic Geometry seminar at University of Warsaw, Warsaw, Poland, 18/04/2013**.**
7. **Attended** *CWI Lexures: Discrete Mathematics and Optimization* at CWI, Amsterdam, The Netherlands, 25/04/2013.
8. **Attended** *Syzygies in Berlin* at Free University of Berlin, Berlin, Germany, 27/05/2013-31/05/2013.
9. **Attended** *Combinatorial structures in Algebra and Topology* at University of Osnabrück, Osnabrück, Germany, 03/07/2013-06/07/2013.

IV – RESEARCH EXCHANGE PROGRAMME (REP)

1. Polytechnic University of Catalonia (UPC), Department of Applied Mathematics I, Barcelona, Spain, 17/02/2013-23/02/2013.
Local scientific coordinator: Prof. Francesc Planas-Vilanova.

During my first visit I had cooperation with Prof. Planas-Vilanova with regards to lattice ideals, including primary decompositions and binomial generating sets of such ideals. Moreover, I gave a talk at the Algebraic Geometry Seminar, which is organized jointly by the University of Barcelona and the Polytechnic University of Catalonia. This research presentation was devoted in presenting a number of scientific results obtained during the ERCIM fellowship.

2. University of Warsaw, Faculty of Mathematics, Informatics and Mechanics, Warsaw, Poland, 15/04/2013-20/04/2013.
Local scientific coordinator: Prof. Jaroslaw Wisniewski.

During my second visit I had the opportunity to discuss with other researchers about toric varieties. I also presented part of my post-doctoral research under the framework of the Algebraic Geometry seminar, which takes part at the University of Warsaw.