<table>
<thead>
<tr>
<th>First name / Family name</th>
<th>Anargyros Katsampekis</th>
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<tr>
<td>Nationality</td>
<td>Greek</td>
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<td>Name of the <em>Host Organisation</em></td>
<td>CWI</td>
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<tr>
<td>First Name / family name of the <em>Scientific Coordinator</em></td>
<td>Monique Laurent</td>
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<td>Period of the fellowship</td>
<td>01/10/2012 to 30/09/2013</td>
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I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

My research activity during the ERCIM fellowship at CWI was concentrated on two directions:

1. Describing the structure of the universal Gröbner basis of an ideal generated by diagonal 2-minors and
2. Computing the exact value of the binomial arithmetical rank of a lattice ideal.

The above mentioned activities are briefly analysed below.

1. Describing the structure of the universal Gröbner basis of an ideal generated by diagonal 2-minors.

My approach for this problem involved toric ideals, a well-studied class of binomial ideals. Since any ideal \( P_\mathcal{G} \) generated by diagonal 2-minors is also a toric ideal, the first step was to find a vector configuration \( \mathcal{G} \) such that \( P_\mathcal{G} \) equals the toric ideal associated to \( \mathcal{G} \). Next, I used this configuration to prove that if \( \mathcal{G} \) is bipartite then, \( P_\mathcal{G} \) is a unimodular toric ideal. As a consequence, for every bipartite graph \( \mathcal{G} \) the initial ideal of \( P_\mathcal{G} \) is generated by squarefree monomials. Moreover, the universal Gröbner basis of \( P_\mathcal{G} \) is determined by the circuits of the toric ideal associated to \( \mathcal{G} \), when \( \mathcal{G} \) is a bipartite graph. To find the circuits of the above toric ideal, I studied the problem of when the ideal \( P_\mathcal{G} \) can be realised as the toric ideal associated to a finite simple graph. I proved that this is exactly the case when every connected component of \( \mathcal{G} \) has at most one cycle. Furthermore, I showed that:
   (a) If \( \mathcal{G} \) is a tree, then the universal Gröbner basis consists of all binomials which correspond to even cycles of the prism of \( \mathcal{G} \).
   (b) If \( \mathcal{G} \) is a connected bipartite graph with exactly one cycle, then the universal Gröbner basis consists of all binomials which correspond to even cycles of an appropriate graph \( \mathcal{H} \).

Finally, I explicitly calculated the number of elements and the maximum degree in the universal Gröbner basis of \( P_\mathcal{G} \), when \( \mathcal{G} \) is either a star graph or a path graph.

2. Computing the exact value of the binomial arithmetical rank of a lattice ideal.

Lattice ideals arise naturally in problems from diverse areas of mathematics. This class of ideals includes also the equiquit ideals and the ideals generated by diagonal 2-minors. A basic problem in Commutative Algebra is to determine the binomial arithmetical rank of a lattice ideal. To deal with this problem I considered the indispensable monomials of a lattice ideal \( I_L \) and introduced the simplicial complex \( \Gamma_L \). Furthermore, I used matchings in simplicial complexes to provide a lower bound for the binomial arithmetical rank of a lattice ideal. This lower bound was computed in two cases:
   (a) \( I_L \) is a toric ideal associated to a number of graphs, including the wheel graph and a weakly chordal graph.
   (b) \( I_L \) is a determinantal ideal generated by the 2x2 minors of a certain matrix of indeterminates.

In both cases the binomial arithmetical rank equals the minimal number of generators of
the lattice ideal.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP


Abstract. Let $G$ be a simple graph on the vertex set $\{1, \ldots, n\}$. An algebraic object attached to $G$ is the ideal $P_G$ generated by diagonal 2-minors of an $n \times n$ matrix of variables. In this paper we first provide some general results concerning the ideal $P_G$. It is also proved that if $G$ is bipartite, then every initial ideal of $P_G$ is generated by squarefree monomials. Furthermore, we completely characterize all graphs $G$ for which $P_G$ is the toric ideal associated to a finite simple graph. As a byproduct we obtain classes of toric ideals associated to non-bipartite graphs which have quadratic Gröbner bases. Finally, we provide information in certain cases about the universal Gröbner basis of $P_G$.


Abstract. To any lattice $L \subset \mathbb{Z}^m$ one can associate the lattice ideal $I_L \subset K[x_1, \ldots, x_m]$. This paper concerns the study of the relation between the binomial arithmetical rank and the minimal number of generators of $I_L$. We provide lower bounds for the binomial arithmetical rank and the $A$-homogeneous arithmetical rank of $I_L$. Furthermore, in certain cases we show that the binomial arithmetical rank equals the minimal number of generators of $I_L$. Finally we consider a class of determinantal lattice ideals and study some algebraic properties of them.

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

4. Talk Circuits of vector configurations and the binomial arithmetical rank of toric ideals of graphs. Algebraic Geometry seminar at Polytechnic University of Catalonia, Barcelona, Spain, 22/02/2013.
5. Attended 4th SDP days at CWI, Amsterdam, 21/03/2013-22/03/2013.
VI – RESEARCH EXCHANGE PROGRAMME (REP)

1. Polytechnic University of Catalonia (UPC), Department of Applied Mathematics I, Barcelona, Spain, 17/02/2013-23/02/2013.
Local scientific coordinator: Prof. Francesc Planas-Vilanova.

During my first visit I had cooperation with Prof. Planas-Vilanova with regards to lattice ideals, including primary decompositions and binomial generating sets of such ideals. Moreover, I gave a talk at the Algebraic Geometry Seminar, which is organized jointly by the University of Barcelona and the Polytechnic University of Catalonia. This research presentation was devoted in presenting a number of scientific results obtained during the ERCIM fellowship.

2. University of Warsaw, Faculty of Mathematics, Informatics and Mechanics, Warsaw, Poland, 15/04/2013-20/04/2013.
Local scientific coordinator: Prof. Jaroslaw Wisniewski.

During my second visit I had the opportunity to discuss with other researchers about toric varieties. I also presented part of my post-doctoral research under the framework of the Algebraic Geometry seminar, which takes part at the University of Warsaw.