




# Scientific Report

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| First name / Family name | Giorgi Baghaturia |
| Nationality | Georgian |
| Name of the *Host Organisation* | University of Warsaw |
| First Name / family name of the *Scientific Coordinator* | Miroslaw Lachowicz |
| Period of the fellowship | 11/04/2013 to 10/04/2014 |

# I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

I was selected as an ERCIM "Alain Bensoussan" Fellow in late 2012. My fellowship started on April 2013 and I was affiliated with the Institute of Applied Mathematics and Mechanics of the Faculty of Mathematics,Informatics and Mechanics in the University of Warsaw. The Faculty, with almost 250 faculty members and PhD students,was invited to join in 2007 the European Research Consortium for Informatics and Mathematics (ERCIM) as a full member.

I was assigned to collaborate with the member of The Mathematical Physics Equations group, Professor Zbigniew Peradzynski by my Scientific coordinator and the director of the Institute of Applied Mathematics and Mechanics, Professor Miroslaw Lachowicz.

Main activity of The Mathematical Physics Equations group is to find existence and regularity of solutions to nonlinear partial differential equations and systems of equations, existence of entropy solutions to nonlinear conservation laws, reaction diffusion equations, free boundary problems, asymptotics, existence of attractors as well as applications in biology, fluid dynamics, plasma and growth of crystals.

Before coming to the University of Warsaw, my research interests concerned with studying of vaious initial and characteristic problems for non-linear Partial Differential Equations of mixed type. In collaboration with Professor Peradzynski I started to work on topics quite different from those I have mentioned, but I also continued to investigate mixed type equations, especially some singular cases regarding Initial problems posed for Quasi-linear PDE-s.

During my stay in Warsaw we studied the following partial differential equation

$$\sum_{i,j=1,..n} g_{ij}(Du)\frac{\partial u}{\partial x^i \partial x^j} = 0\,, \qquad (1)$$

where $g(\mathrm{Du})$ is nonsingular indefinite nxn matrix with entries depending on first derivatives of the scalar function $u(x)$. So we deal with a nonlinear hyperbolic equation of second order. By introducing new variables $U_i = \frac{\partial u}{\partial x^i}$ we arrive at the equivalent system of first order quasi-linear equations

$$\partial x_j U_i = \partial x_i U_j \qquad \sum_{i,j=1,..n} g_{ij}(U)\partial x_i U_j \qquad (2)$$

It is well know that such a system has solutions representing double waves solutions . Such solutions can be thought of as resulting from the interactions of two simple waves ( known also as Riemann waves). More precisely;

1. if $U = f_1(R^1)$ , $U = f_2(R^2)$ are two different solutions to the ordinary differential equation

$$\sum_{i,j} g_{ij}(U)\frac{dU_i}{dR}\frac{dU_j}{dR} = 0$$

such that $f_1(0) = f_1(0)$ then they represent the images of two simple waves. For example the the first simple wave is defined by

$$U = f_1(R^1) \text{ and } \quad dR^1 = \xi \sum_i \dot{f}_{1i}(R^1)dx_i$$

where " $\cdot$ " denotes the differentiation.
The general solution for the equation for $R^1(x)$ can be expressed by the following implicit formula

$$R^1 = \varphi(\dot{f}_1(R^1)\cdot x)$$

where $\dot{f}_1(R^1)\cdot x$ is the scalar product of $\dot{f}_1(R^1)$ and $x$.

2. Given $f_1(R^1)$, $f_2(R^2)$ one proves that there exists a function $f(R^1,R^2)$ which is a solution to the system

$$U_{R^1R^2} = \alpha_2^1 U_{R^2} + \alpha_1^2 U_{R^1}$$

satisfying the following Goursat conditions: $f(R^1,0) = f_1(R^1)$, $f(0,R^2) = f_2(R^2)$. This solution is defined in general locally ( for some $|R^1| \leq a$, $|R^2| \leq b$ ) . The function $f$ defines the two-dimensional range (hodograph) of the double wave solution provided that

$$\alpha_j^i = -g(U_{R^1},U_{R^2})^{-1}\, g_{R^j}(U_{R^i},U_{R^i})$$

Solution U(x) itself is defined by

$$U = f(R^1,R^2) \quad \text{and} \quad dR^i = \sum_i \xi^i f_{R^i}\, dx_i$$

In spite of the fact that the system for $R^1, R^2$ is over-determined it has solution depending on two functions of single variable ( defining the profiles of waves). The coefficients $\alpha_2^1, \alpha_1^2$ define the strength of the interaction. If they vanish then the waves are not interacting and the double wave solution represented as the linear superposition of two (in general nonlinear) waves
The following questions were addressed in our research :

1. Given the coefficients $\alpha_j^i(R^1,R^2)$ construct all possible equations of type (2) having these coefficients as coefficients of interactions of double waves.
2. Construct all possible equations of type (2) having solutions in the form of triple waves ( then $U = f(R^1,R^2,R^3)$ )
3. In general we can ask the question concerning existence of k- waves if $n \geq k$.
4. Find all possible equations of type (2) such that interaction of k simple waves is represented as k-wave solution.

In particular we may ask what is the general form of the system (2) , so that there are k-waves representing the linear superposition of k different waves. As an example, it is known that the system of ideal gas-dynamics has solution in the form of three non-interacting Riemann waves.
The results of our research are now prepared for publication.

## II – PUBLICATION(S) DURING YOUR FELLOWSHIP

***Title*:** On k-tuple waves for the second order quasilinear hyperbolic equation

***Authors*:** Baghaturia G., Peradzynski Z.

***Status*:** **in preparation for publication**

***Abstract.*** Special solutions in the form of k-tuple waves for a quasi-linear hyperbolic equation of second order are considered. These solutions can be interpreted as resulting from the interaction of single waves (Riemann waves).We specify the general form od such an equation which admits a sufficiently large set of k-tuple vave solution.

## III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

I attended the following seminars:

1.Weekly seminarium in Biomathematics and the game theory (Faculty of Mathematics Informatics and Mechanics, University of Warsaw).

2.Weekly Seminarium of Equations of Mathematical Physics ((Faculty of Mathematics Informatics and Mechanics, University of Warsaw).

3. Joint Seminar of Scientific Computing and Multiscale Dynamics groups at Centrum Wiskunde & Informatica (CWI), Amsterdam, February 19**,** with Presentation "**Cauchy and Goursat problems for a second order quasi-linear equation of mixed type".**

I also attended non-scientific ABCDE seminar III organized by ERCIM, October 31, November 1. Athens,Greece (with a brief presentation about my fellowship activity).

I attended the following International Conference:

"Mathematics, Mechanics and Modelling, a tribute to Zbigniew Peradzynski", Bedlewo,Poland, organized by the Faculty of Applied Mathematics AGH University of Science and Technology.September 22-27, with a presentation "On some hyperbolic problems for a second order quasi-linear equation of mixed type".

Abstract. The Cauchy and Goursat problem for one quasilinear second order hyperbolic

equation with possible order and type degeneration is considered. Sufficient condition for the existence of classical solutions are obtained.

# IV – RESEARCH EXCHANGE PROGRAMME (REP)

In the framework of research exchange programme I have visited two ERCIM institutes:

**INRIA Paris-Rocquencourt.**
Research team "BANG".
*Domaine de Voluceau*
*Rocquencourt - B.P. 105*
*F78153 Le Chesnay cedex.*

*UPMC, 4 place Jussieu, Paris:*
*Laboratoire Jacques-Louis Lions*
*Université Pierre et Marie Curie*
*Boîte courrier 187*
*F75252 Paris cedex 05*
**Visiting period: 13 – 23 November, 2013.**
Name of contact person: **Benoît Perthame,** Professor, head of Laboratoire Jacques-Louis Lions (LJLL). e-mail:
benoit.perthame@ljll.math.upmc.fr
Tel. (33 1) 44 27 85 18

**Centrum Wiskunde & Informatica (Center for Mathematics and Computer Science)** in Amsterdam.
Science Park 123
1098 XG
P.O. Box 94079
1090 GB Amsterdam,
The Netherlands

Visiting period: 16 –22 February, 2014.
Contact person:
**Dr.ir. Jeroen A.S. Witteveen**,
Scientific Staff Member,
Scientific Computing group, Energy theme,
Center for Mathematics and Computer Science (CWI),
Amsterdam, The Netherlands,
e-mail address: djeroen@cwi.nl
room L123
Tel.: +31(0)20 592 4085