Scientific Report

First name / Family name	Antti Haimi
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Period of the fellowship	1/9/2013 – 31/1/2015

I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

My scientific activity during the fellowship period has concentrated on the intersection of time-frequency and complex analysis. In particular, I have been analysing properties of so called short-time Fourier transform. The difference between the short-time Fourier transform and the standard one is that with the former, one aims at capturing the information about frequencies which are present in the signal at any given moment. It is well known that it is very difficult to extract this sort of information from standard Fourier representation. Localization around different time points is attained by multiplying the original signal by shifts of a fixed function. This function is often called a *window*. It is a fundamental question in time-frequency analysis to describe discrete sets in the time-frequency plane such that the signal is fully determined given the values of the short-time Fourier transform on those points. The nature of these sets depends intimately on the window used. If the time-frequency set is a lattice, it is known that the density has to be greater than one. As for the sufficiency of this condition, so far there has mostly been results concerning specific windows, most notably the Gaussian. In a joint work with Karlheinz Gröchenig and Jose Luis Romero from University of Vienna, we show that necessary condition is also sufficient for a large variety of windows given that the time-frequency lattice has a rational density. When the window is a Hermite function, this result has interesting consequences in complex analysis, more precisely concerning uniquencess sets of so called polyanalytic functions. An interesting fact is that at the moment, it is not known how such a result could be proved using only complex analytic methods.

We think that our results can shed light to another problem in this area, namely description of sampling sets. By this we mean discrete sets in the time-frequency plane that so that the values of the short-time Fourier transform can be used not only to determine the signal completely but also to estimate its size. Sampling sets of the short-time Fourier transform are frequently used in many fields, for example geophysics, medical imaging and telecommunications.

I have also been working on a problem in complex analysis concerning generalization of Hörmander's L^2 estimates for the dbar-operator to higher order powers of this operator. This study is related to the vector-valued short-time Fourier transform where a vector of first Hermite functions is used as a window. This topic has potential implications in multiplexing, a widely used method in telecommunications where one aims to capture the content of several signals into one representation. In this area, I have obtained results which show that completely new conditions on the weight, which are very different from those that can be obtained by iterating Hörmander's standard estimate, can appear.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

K. Gröchernig, A. Haimi, J. L. Romero. Completeness of Gabor families, in preparation.

A.Haimi, H. Hedenmalm. Asymptotic expansion of polyanalytic Bergman kernels, J. Funct. Anal. 267 (2014), no. 12, 4667-4731.

Abstract:

We consider the q-analytic functions on a given planar domain Ω, square integrable with respect to a weight. This gives us a q-analytic Bergman kernel, which we use to extend the Bergman metric to this context. Polyanalytic Bergman spaces and kernels appear naturally in time-frequency analysis of Gabor systems of Hermite functions as well as in the mathematical physics of the analysis of Landau levels.

We obtain asymptotic formulae in the bulk for the q -analytic Bergman kernel in the setting of the power weights e^{-2mQ}, as the positive real parameter m tends to infinity. This is only known previously for q=1, by the work of Tian, Yau, Zelditch, and Catlin. Our analysis, however, is inspired by the more recent approach of Berman, Berndtsson, and Sjöstrand, which is based on ideas from microlocal analysis. We remark here that since a q-analytic function may be identified with a vector-valued holomorphic function, the Bergman space of q -analytic functions may be understood as a vector-valued holomorphic Bergman space supplied with a certain singular local metric on the vectors. Finally, we apply the obtained asymptotics for q=2 to the bianalytic Bergman metrics, and after suitable blow-up, the result is independent of Q for a wide class of potentials Q.

A. Haimi. Bulk asymptotics for polyanalytic correlation kernels, J. Funct. Anal. 266 (2014), no.5, 2083-3133.

Abstract:

We study reproducing kernels of weighted spaces of polyanalytic polynomials on the complex plane. The results include a universality result concerning local blow-ups of the kernels near so called bulk points as well as an off-diagonal decay estimate.

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

"Complex Analysis and Related Topics" in St. Petersburg, April 14-18 2014.

Eleventh Advanced Course on Complex Analysis and Operator Theory", in Sevilla, 16-18 June 2014.

IV – RESEARCH EXCHANGE PROGRAMME (REP)

I spent 18-30 October 2013 at UPC in Barcelona. The host of the visit was professor Jaume Amoros from department of mathematics. This visit was very important as it allowed me to interact with local experts in analysis and also to present my research in the local seminar.

The second research exchange visit was to University of Malaga on 23-27 June 2014. The host at the mathematics department was professor José Angél Peláez. We had many interesting mathematical discussions and shared some thoughts on our common topic of interest, weighted Bergman spaces.