ERCIM "ALAIN BENSOUSSAN" FELLOWSHIP PROGRAMME

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Scientific Report

First name / Family name	Nikolai Osipov
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Name of the *Host Organisation*	NTNU
First Name / family name of the *Scientific Coordinator*	Yurii Lyubarskii
Period of the fellowship	01/10/2014 to 30/09/2015

I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

- A publication unifying Bellman function approach in harmonic analysis has been prepared and published. This is a joint work with P.Ivanisvili, D.M.Stolyarov, V.I.Vasyunin, and P.B.Zatitskiy. We have managed to get our results due to a deep connection between harmonic analysis, probability theory (martingales), and differential geometry.
- A publication about an analogue of Rubio de Francia's inequality for the Walsh system has been prepared. In order to obtain this result, some probabilistic technique (dyadic martingales) and interesting combinatorial considerations have been employed.
- A publication generalizing Rubio de Francia's inequality to Triebel–Lizorkin and Besov spaces has been almost prepared. This is a joint work with Eugenia Malinnikova (NTNU). Our results outline an overall picture in which context we raise and answer a subtle question concerning the presence or absence of the rotations in the definition of Rubio de Francia's operators.
- Many talks have been made in NTNU and other places (see Part III and IV of this report).
- Also I would like to express my gratitude to editorial board of ERCIM News for publication of the article on a side project in which I am participating: http://ercim-news.ercim.eu/en102/r-i/virtual-prediction-markets-in-medicine

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

1. P.Ivanisvili, N.N.Osipov, D.M.Stolyarov, V.I.Vasyunin, P.B.Zatitskiy, *Sharp estimates of integral functionals on classes of functions with small mean oscillation*, **accepted** in Compt. Rend. Math. (see also http://arxiv.org/abs/1412.4749)

Abstract: We unify several Bellman function problems into one setting. For that purpose we define a class of functions that have, in a sense, small mean oscillation (this class depends on two convex sets in R^2). We show how the unit ball in the BMO space, or a Muckenhoupt class, or a Gehring class can be described in such a fashion. Finally, we consider a Bellman function problem on these classes, discuss its solution and related questions.

2. N.N.Osipov, *Littlewood–Paley–Rubio de Francia inequality for the Walsh system*, **preprint**, http://arxiv.org/abs/1412.2266

Abstract: Rubio de Francia proved the one-sided Littlewood--Paley inequality for arbitrary intervals in L^p, $2 \le p < \infty$. In this article, such an inequality is proved for the Walsh system.

3. E.Malinnikova, N.N.Osipov, *Two types of Rubio de Francia operators on Triebel–Lizorkin and Besov spaces*, **in preparation**

Abstract: Generalizations of Rubio de Francia's inequality for Triebel–Lizorkin and Besov spaces have been obtained. Our results outline an overall picture in which context we raise and answer a subtle question concerning the presence or absence of the rotations in the definition of Rubio de Francia's operators.

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

1. *Complex and harmonic analysis seminar*, NTNU, Dep. of Math., title of my talk is "*Some new results concerning the Littlewood–Paley–Rubio de Francia inequality*".
2. *Complex and harmonic analysis seminar*, NTNU, Dep. of Math., title of my talk is " *Littlewood–Paley–Rubio de Francia inequality for* BMO*-space and for Hölder classes of smooth functions*".
3. *24th St.Petersburg Summer Meeting in Mathematical Analysis*, June 25, 2015 — June 30, 2015, St.Petersburg/Russia, title of my talk is "*Two types of Rubio de Francia operators on Besov and Triebel–Lizorkin spaces*" (joint work with Eugenia Malinnikova).
4. *Workshop SAFSI 2014,* December 1, 2014 — December 5, 2014, St.Petersburg/Russia, title of my talk is " *Littlewood–Paley–Rubio de Francia inequality for the Walsh system*".

IV – RESEARCH EXCHANGE PROGRAMME (REP)

Place and organization: Inria, France, Sophia Antipolis.
Dates: June 2, 2015 — December 8, 2014.
Local scientific coordinator: Laurent Baratchart.

I have made a talk for Professor Laurent Baratchart and his group. The talk title is "*Littlewood–Paley–Rubio de Francia inequality for the Walsh system*". I have also discussed various studies conducted by the members of Laurent Baratchart's group. One of them (Dmitry Ponomarev) have expressed interest in the initiative project mentioned above and readiness to participate in it.